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FAST MAPPING OF SUBSURFACE SOILS, WATER AND CONTAMINANTS USING MULTIPLE ELECTROMAGNETIC SENSORS

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Fast Mapping of Subsurface Soils, Water and Contaminants Using Multiple Electromagnetic Sensors

Final Report

Project Number 2006-0131

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Introduction

Mobile electromagnetic instruments are often used to measure the electrical conductivity of the soil. Soil conductivity is related to a number of other soil properties, and soil surveyors have used conductivity data to estimate a number of subsurface soil properties including salinity, soil moisture, contaminants, soil type, and soil depth. The development of more accurate and detailed soil conductivity maps can provide a number of benefits to the agricultural sector:

Crop Yields
If farmers have more detailed maps of the soil properties that affect crop yield, they can apply management practices that can reduce costs and/or increase yields. Yields may be increased if crop varieties can be matched to production-related soil properties such as water content, topsoil depth, soil nutrient levels and salinity. Variable rate fertilizer application and variable rate seeding based on accurate soil maps can reduce input costs.

Salinity Mapping
Conductivity measurements correlate well with salinity. Accurate salinity maps with good horizontal and depth resolution can improve the targeting and management of fresh and saline groundwater. High risk areas can be targeted for salinity treatments.

Site Selection of Wells, Dugouts and Irrigation Projects
Conductivity surveys have been used to locate fresh water sources. Drilling costs can be reduced if more accurate soil maps are used to select sites for wells and dugouts.

Placement of On-Farm Tests
Farmers may want to test two or more varieties of a crop, and plant each variety in the same field. Differences in yield may be due to differences in soil properties between the test areas. If farmers have accurate soil maps, they can ensure that each variety is planted in the same type of soil. This will reduce the effect of extraneous factors on their interpretation of test results.

Ranching
Conductivity surveys can be used to predict variations in soil depth. The resulting soil depth maps can be used to select sites for rangeland monitoring and management of grazing.

Site Selection of Waste Lagoons
The optimum placement of waste containment systems depends on a number of factors, including horizontal changes in soil types, the presence of paleochannels and the location of groundwater. Accurate soil maps can prevent costly remediation efforts due to less than optimal locations for waste barriers.

Monitoring Waste Containment Systems
Repeated conductivity surveys can be used to monitor seepage from waste containment systems. Repeated conductivity measurements can also measure the amount of water content in soil barriers used to contain waste lagoons.

Monitoring Contaminants from Other Sources
Nutrient buildups may occur at feedlot or manure handling sites. Feedlot manure that is applied to fields may change soil nitrogen balances. Repeated surveys can monitor ionic concentrations on or near the soil surface.

Site Selection of Dams
The construction of dams for irrigation and water control projects requires a good knowledge of subsurface conditions. The placement of dam foundations depends on various subsurface soil properties. Accurate three-dimensional soil maps can reduce the number of drill holes required to find suitable locations for dam construction.

Research Objectives

Our research objectives were to:

- Develop fast, accurate and inexpensive field methods to map subsurface conductivity at the field level.
- Develop new interpretation methods to estimate conductivity changes with soil depth.
- Predict subsurface soil properties, such as salinity and soil moisture from the conductivity measurements.

We have developed and applied a variety of new field survey methods and interpretation techniques to realize these objectives. The specific methodology and results for each technique are discussed in the following sections of this report.

Summary of Findings

Staff at EcoTech Research worked with scientists and technicians at PFRA's Maxwell Laboratory in Regina and the Saskatchewan Ministry of Agriculture's Irrigation Development Branch in Outlook, Saskatchewan to develop fast methods to map subsurface conductivity and estimate soil properties.

One research objective was to increase the amount of conductivity information gained in a survey by simultaneously recording data from a Geonics EM-38DD ground conductivity meter, and a Geonics EM-31 ground conductivity meter. Several issues had to be resolved:

- Would the instruments interfere with one another?
- Could the instruments be attached to a vehicle without the metal in the vehicle interfering with the conductivity readings?

Our field tests show that the EM-38DD and the EM-31 instruments can be placed quite close to one another without serious interference. The instruments can be placed one behind the other, parallel to one another, or perpendicular to one another.

- If both instruments are parallel to one another, the level of interference between the instruments will be small as long as the separation is 0.8 meters or greater.
- If the instruments are perpendicular to one another, the level of interference will be small as long as the instruments are oriented so that the EM-31 receiver is closest to the EM-38DD transmitter, and the instruments are separated by 0.74 meters.

These results indicate that it is possible to mount both instruments on a single cart or sled and simultaneously record data from both instruments. Survey times and costs can be reduced by over fifty percent if a dual instrument survey is carried out rather than two separate surveys each using a single instrument. Our results allow survey operators several different instrument orientations to choose from when they design their instrument carts or sleds.

Survey time and costs can be further reduced if the instruments can be attached directly to an all terrain vehicle instead of being towed in a sled or cart. Most of this saving is due to reduced equipment setup and take down time. There are also some advantages to the ATV operator if the instruments are close to vehicle. Our experiments with an ATV used by PFRA staff in Regina showed that the EM-38DD can be safely placed in front of the ATV. The instrument can be set at any survey height, and a simple correction can be made to remove the effects due to the conductivity of the ATV.

The EM-31 can also be placed on a boom to the side of the ATV. Interference was lowest when the EM-31 transmitter was closest to the front of the ATV. Moving the EM-31 away from the ATV and displacing it back from the ATV reduces the amount of interference. Preliminary results indicate that the EM-31 can be placed at any height, and that a simple correction can be used to remove the effect of interference from the ATV. However more field tests need to be done to confirm this.

In order for a multi-instrument survey to be practical, we had to create a hardware interface between the survey computer and the EM-38DD, EM-31 and GPS instruments. We also had to create a computer program to record data simultaneously from the three instruments. The computer program we created allows the ATV operator to view a computer screen that displays the survey path and survey data in real time. The program also warns the operator if there is a problem with the instruments. At the end of a field survey the operator can quickly create contour maps of the GPS and conductivity data. The operator can determine if any areas were missed or if there were problems with the data. This allows the operator to add more survey lines if necessary. This contouring feature makes it possible for survey operators to deal with data acquisition problems right away, rather than going back to the lab, checking the data, and then returning to the field to re-do part of the survey.

Ground conductivity surveys have been carried out where the instrument measurements are made either at the surface or above the surface. There is no standard surveying height. We analyzed the effect of raising the instruments above the ground on the survey data. We discovered that there are several effects.

Raising the height of the instrument increases the effective depth of penetration of the instrument. If a known target, such as a water source or pollution plume is expected to be at a certain depth, the operator can increase the height of the meter so that data will be collected from this depth.

Raising the height of the instrument reduces the amplitude of the apparent conductivity measurement. This effect is more pronounced for the EM-38DD than for the EM-31. In many cases interpretation methods can be adjusted to account for this change in amplitude. For example, apparent salinity levels can still be calculated from EM-38DD data even if the instrument is above the ground.

Lower conductivity readings due to increased instrument height can sometimes be a problem when we want to detect lateral changes in soil conductivity. In most field situations operators will be able to distinguish conductivity differences from contour maps of soil conductivity regardless of the instrument height. The only time there may be a problem is when two adjacent soils have conductivities that are close in amplitude. We have devised an alternative interpretation method that contours conductivity ratios in a field. This measure highlights conductivity differences well, and can be used even when the instrument readings are taken 1 meter or more above the surface.

Some researchers have developed methods to estimate surface measurements from above surface EM-38 measurements. Our analysis shows that this method only works if there are no changes in conductivity down to the effective penetration depth of the EM-38 instrument. Modelling results and field tests indicate that these correction formulas can give highly inaccurate results when soil conductivity varies with depth.

The situation is less severe when the EM-31 instrument is above the ground. If conductivity is constant down to the effective depth of the instrument, and if the EM-31 is less than 0.4 meters above the surface, then the conductivity measurements will underestimate surface readings by about 2%. In such cases no corrections need to be made. If the instrument is set higher than this, or if the instrument is in the horizontal orientation, then corrections may be made.

We also developed correction formulas for the EM-31 using the assumption of uniform conductivity with depth. We used computer models and field tests to check the accuracy of these corrections when the soil was not uniformly conductive with depth. We determined that even at an instrument height of 1 meter, over 90% of the corrections made to the vertical dipole reading were accurate to within 5% of the actual ground value.

We developed interpretation techniques to estimate the conductivities and thickness of two layer and three layer soil profiles. The added information provided by surveying with an EM-38DD and EM-31 simultaneously allows us to make inferences about changes in soil conductivity with depth. We developed several numerical techniques to estimate soil properties with depth using different instrument combinations and heights. We found that if the depth to the first layer is available to us, then accurate analytic formulas can predict the conductivities in the two layers. If we know the depths of the first two layers, we can accurately predict the conductivities in these layers and in the third layer.

Many soil conductivity surveys involve a single EM-38DD instrument and all measurements are made on the surface. If we have no knowledge of the depth to the first layer, we cannot create formulas to estimate the conductivities of each layer. We have developed a statistical method to estimate the conductivity of the top soil horizon based solely on data from a single EM-38DD where readings are taken at surface level.

Soil surveys collect GPS as well as conductivity data. The GPS data is used to locate each reading so that contour and other conductivity maps can be made. The altitude data can also be used to create topographic maps of an area. We created a computer program that uses all three GPS components to calculate various topographic attributes of a field, including slope, curvature, flow direction and other parameters. In addition the program also calculates the Topographic Wetness Index for a field. This program may be useful in predicting soil moisture and topsoil thickness from the topography. While this method is not useful in flat areas, it may be of some use in predicting moisture conditions in areas where there are changes in elevation. The computer program is available for use, but we have not had a chance to test the results with data from the prairies.

Interference between the EM-31 and EM-38DD instruments

Soil conductivity surveys are typically carried out using a single instrument. The Geonics EM-38DD is commonly used for such surveys (Figure 1). When greater depth penetration is required the EM-31 instrument is used (Figure 2). If we can collect data from both instruments simultaneously, we will have a better understanding of how conductivity varies with depth (McNeill, 1980).

Figure 1: Geonics EM38-DD dual dipole ground conductivity meter (Geonics)

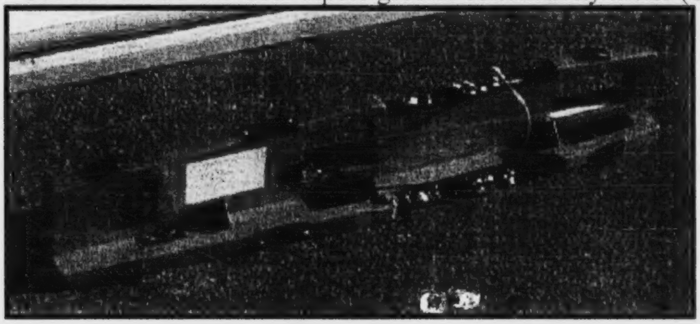
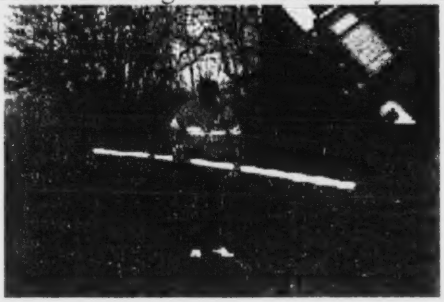
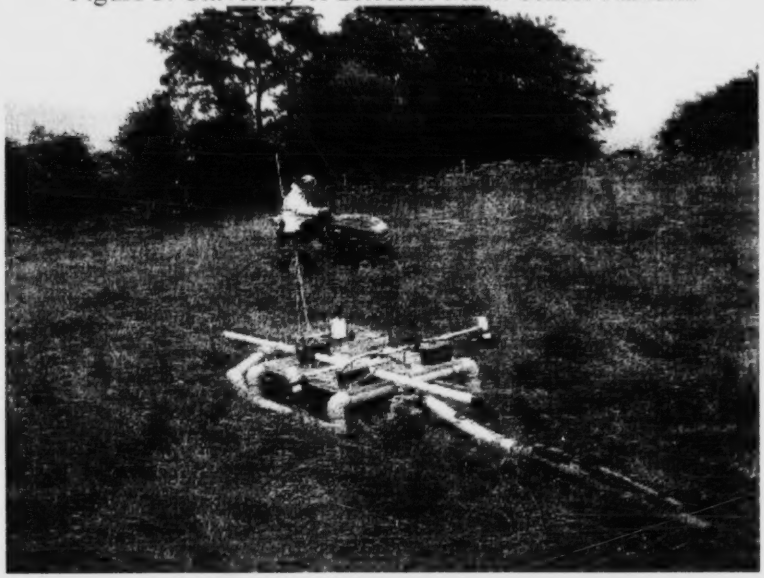


Figure 2: Geonics EM-31 ground conductivity meter (Geonics)



If both instruments are used simultaneously, the transmitted signal from one instrument can interfere with the conductivity measurements made by the second instrument. If this interference cannot be dealt with, then it will be necessary to run a survey twice over the same field using one instrument at a time. This is costly. One of our research objectives was to determine if the two instruments could be placed close enough together so that they could be housed on a sled or a cart. This would reduce survey costs and provide more information to the soil surveyor. Some surveyors have already experimented with multiple-instrument platforms. The following image is of a sled holding an EM-38, EM-31 and other geophysical systems (Figure 3). This experimental system was developed at the University of Leicester. The EM-31 is placed on the sled and the EM-38 is towed behind the sled.

Figure 3: University of Leicester Multi-Sensor Platform

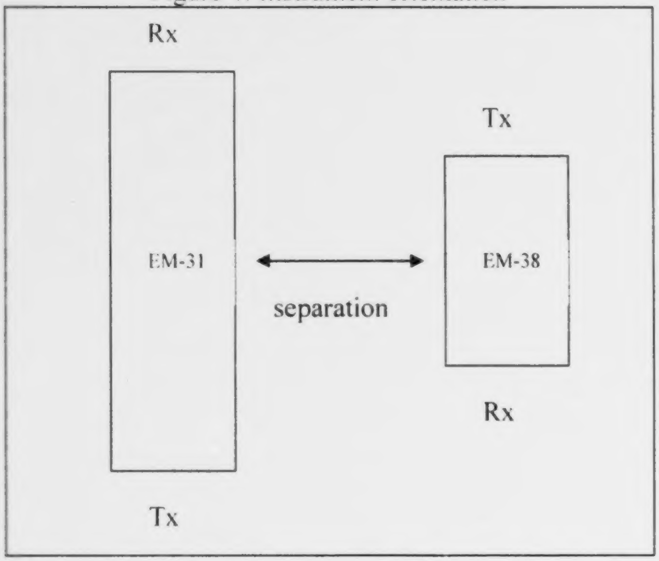


We have tested various geometric configurations of the EM-31 and EM-38DD in order to determine how far apart the instruments must be placed so that interference is minimal. Two sets of measurements were taken. In one case the instruments were placed parallel to each other; in the second case the instruments were at 90° to each other.

Parallel instruments

The first experiment tested the effect of the EM-38DD on readings measured by the EM-31. The EM-31 was placed on the ground and a measurement was made of the conductivity. No other instrument was present when this baseline reading was taken. Subsequently the EM-38DD was placed parallel to the EM-31 at a distance of 0.2 meters. The conductivity readings measured by the EM-31 were recorded. The gap distance between the two instruments was increased by moving the EM38-DD. Several measurements were taken out to a separation of 3.8 meters. In this first experiment the transmitters (Tx) and receivers (Rx) of the two instruments were in opposite directions (Figure 4). A second set of measurements were taken when the two instruments had similar orientations.

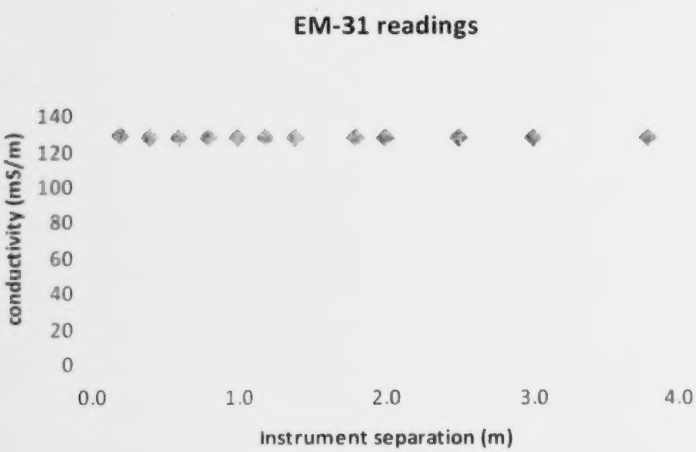
Figure 4: Instrument orientation



Effect of the EM-38DD on EM-31 readings

The presence of the EM-38 had little impact on the measurements taken by the EM-31 (Figure 5). At a separation of 0.2 meters the readings were 127 mS/m, compared to 126 mS/m when the EM-38 was placed at a distance of 3.8 meters (Appendix A). Similar results were achieved when the transmitters and receivers of both instruments were in the same direction. Susceptibility readings were also measured as a function of the instrument separation. Susceptibility values were strongly affected by the presence of the EM-38DD. The susceptibility error was approximately 5% at a separation of 2.5 meters, and increased to over 100% at a separation of 0.2 meters.

Figure 5: EM-31 measurements with separation from the EM-38DD

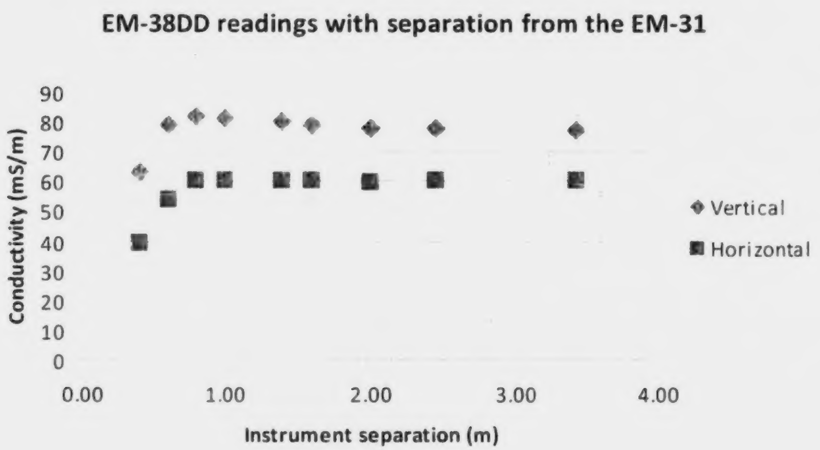


Effect of the EM-31 on EM-38DD readings

In the second experiment we measured the level of interference caused by the EM-31 on the EM-38DD. The methodology was identical to that for the previous experiment. The EM-38DD remained stationary and the EM-31 was moved to different separations. The transmitters and receivers of these instruments were oriented in opposite directions, as in Figure 1 above. The only difference in method was that two conductivity readings were made for the EM-38DD. This instrument consists of two EM-38 conductivity meters where the transmission and separation coils are perpendicular to each other. We measured vertical dipole values (V) and horizontal dipole values (H) for each separation.

The EM-31 has a greater effect on the EM-38DD than vice versa (Figure 6). The vertical component of the EM-38DD is accurate down to a separation of approximately 0.6 meters, while the horizontal component is accurate down to 0.8 meters (see Appendix A).

Figure 6: EM-38DD measurements with separation from the EM-31



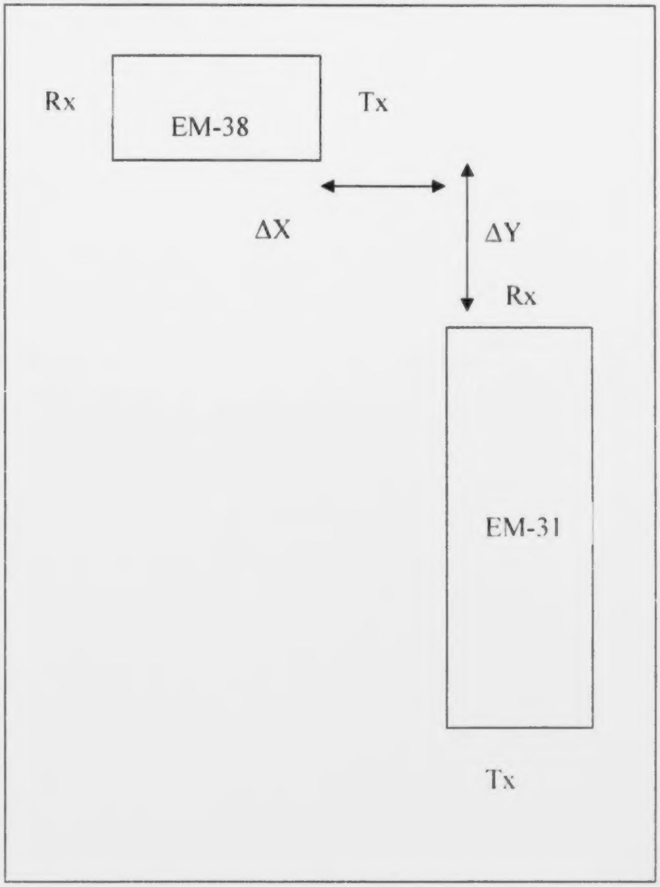
Conclusion

If both instruments are placed in parallel on a sled or a cart, the level of interference between the instruments will be small as long as the separation is 0.8 meters or greater. If the separation is smaller, the EM-31 measurements will be least affected, and the EM-38 horizontal component measurements will be most affected.

Perpendicular instruments

In the next set of experiments we placed the instruments perpendicular to each other, as shown in Figure 4. Several instrument orientations were tested. The orientation shown in Figure 7 has the EM-38DD transmitter (Tx) placed close to the EM-31 receiver (Rx). We refer to this as the R31-T38 orientation. We also tested for interference effects when the orientations were T31-R38, T31-T38 and R38-R31. For each orientation we changed the ΔX separation while leaving the ΔY separation constant at zero meters.

Figure 7: Instrument orientation

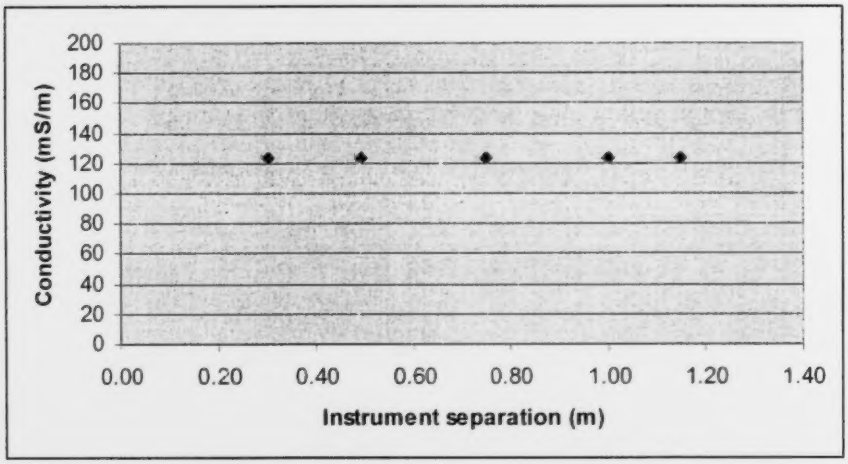


To measure the effect of the EM-31 on EM-38DD readings, we measured the soil conductivity on the ground using the EM-38 when no other instruments were nearby. We then placed the EM-31 close to the EM-38DD and recorded the EM-38DD conductivity readings at different separations of ΔX. ΔY was set to zero. To measure the effect of the EM-38DD on the EM-31 readings, we repeated this procedure with the EM-31 stationary, and the EM-38DD moving.

Effect of the EM-38DD on EM-31 readings

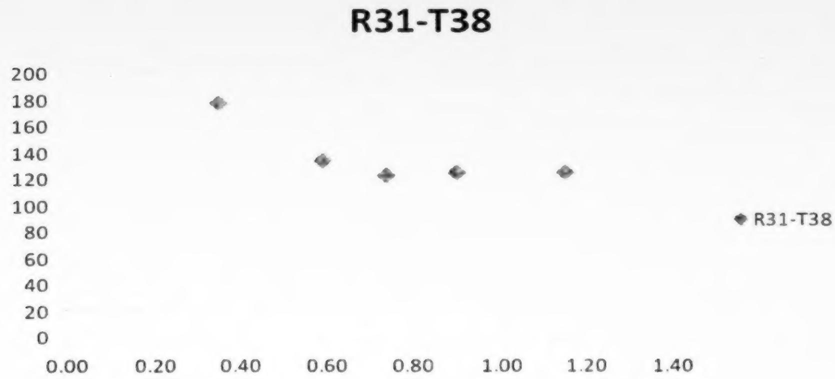
At one test site the EM-31 conductivity reading was 123.5 mS/m when no other instruments were nearby. For three of the possible instrument orientations, there was very little effect on the EM-31 readings even when the instruments were only 0.3 meters apart (Figure 8, Appendix A). Readings were within 1.5 mS/m of the correct value.

Figure 8: EM-31 measurements with separation from the EM-38DD
Orientation T31-T38



Larger interference effects were present when the instruments were in the R31-T38 orientation, where the EM-38DD transmitter was close to the EM-31 receiver. The EM-31 instrument overestimated the conductivity at separation of less than 0.74 meters (Figure 9, Appendix A).

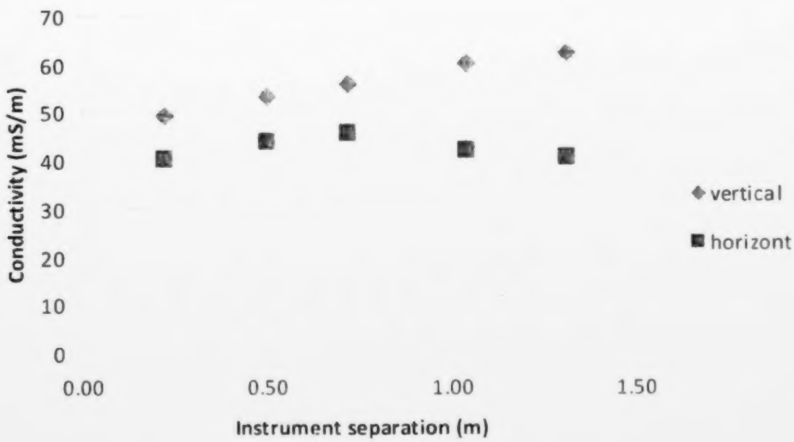
Figure 9: EM-31 measurements with separation from the EM-38DD: R31-T38



Effect of the EM-31 on EM-38DD readings

Due to time constraints we were only able to measure interference effects in two orientations: T31-T38 and R31-T38. In the absence of other instruments the EM-38DD readings were 64 mS/m in the vertical dipole orientation and 40.5 mS/m in the horizontal dipole orientation. There were large interference effects when the transmitter of the EM-31 was close to the transmitter of the EM-38DD (Figure 10, Appendix A).

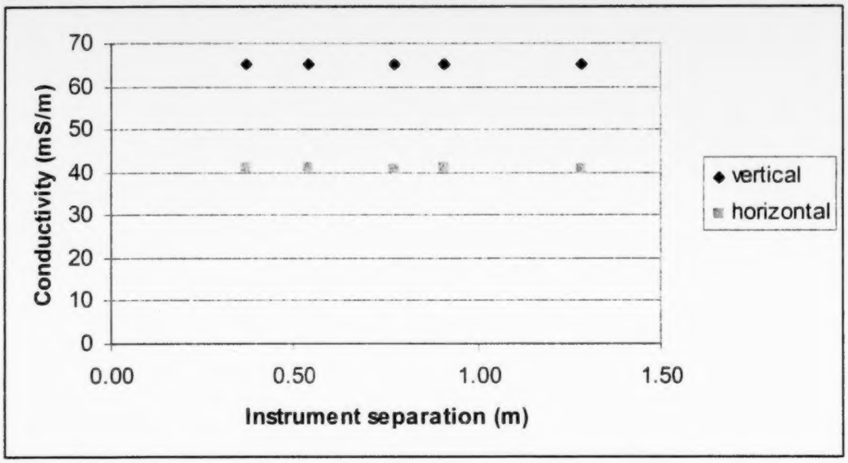
Figure 10: EM-38DD measurements with separation from the EM-31: T31-T38



A separation of 1.3 meters is necessary in order for interference effects from the EM-31 to be acceptable.

Interference effects were much smaller when the EM-38DD was positioned next to the EM-31 receiver, rather than the transmitter (orientation R31-T38). EM-38DD measurements were accurate down to a separation of 0.37 meters (Figure 11).

Figure 11: EM-38DD measurements with separation from the EM-31: R31-T38



Conclusions
It is possible to place both instruments on a sled or cart and position them so their mutual interference is small.

If the instruments are parallel to one another they should be separated by at least 0.8 meters.

If the instruments are oriented in the T31-T38 orientation, there should be a separation of 1.3 meters in the ΔX direction between the two instruments.

If the instruments are oriented in the R31-T38 orientation, there should be a separation of 0.74 meters in the ΔX direction between the two instruments.

Future work
It is likely that the instruments could be moved even closer together if they were in the R31-R38 orientation. Keeping the transmitters as far away from the receivers may result in the lowest instrument interference, but this still needs to be tested.

Instrument separations were only measured in the ΔX direction, while ΔY remained constant. Further work should be done on determining the effect of separation in the ΔY on interference.

Recording data from three instruments simultaneously

There are currently no three instrument systems available on the market, and no commercial software available that allows a user to simultaneously capture data from multiple conductivity meters. One goal of this project was to record data from an EM-31, an EM-38DD and a GPS unit simultaneously. This involved developing both a computer program and the necessary electronics to connect the instruments to a computer. We tested our methods using the equipment provided by PFRA. This consisted of an EM-31, and EM-38DD, and a Holux GR-213U GPS receiver. PFRA employed a Hammerhead XRT laptop computer to record field data. The operating system was Windows XP.

The first step was to connect the three instruments to the computer. The EM-31 and EM-38DD both transmit data via RS-232 ports on each instrument. The data rate is approximately ten readings per second. The ports are configured as follows:

Baud rate	9600
Parity	none
Data bits	8
Stop bit	1

The EM-31 transmits data including conductivity and susceptibility information. The EM-38DD consists of two EM-31 instruments placed at right angles to each other. Geonics has set up the electronics on the EM-38DD so that there is only one data port for both the EM-38 instruments. During a survey the EM-38DD will transmit conductivity data from each instrument.

We attached USB2.0 serial adapters to each Geonics instrument. USB cables connected these instruments to a USB2.0 four port hub. A USB cable was also used to connect the Holux GPS unit to the hub. The hub was then connected to the Hammerhead laptop.

We created a data capture program using Microsoft Visual Basic running under the .NET framework. All programming took place using the Vista operating system. The data capture program will run under Windows XP or Vista. The data capture program is called **ETREMS** (EcoTech Research Electromagnetic Survey System). This program can simultaneously capture data from both Geonics instruments and the GPS unit. The program works even if only one Geonics instrument is present. If an EM-31 is present, the program will capture both the conductivity and susceptibility data.

The ETREMS program will allow the user to input descriptive field information at the start of a survey. It will also check to ensure that the various instruments are connected correctly with the computer. During a survey the program will:

- Provide warning messages if a connection with an instrument is lost.
- Display the track the instruments have followed.
- Display the values that are being recorded.
- Check for bad or missing data.
- Automatically save the recorded data to the computer's hard drive.

At any point during a survey the operator can use the ETREMS program to create contour maps for the GPS altitude and for each data component that was recorded. In the case of an EM-31 and EM-38DD survey, there will be 4 geophysical contour maps that can be created:

- EM-31 conductivity
- EM-31 susceptibility
- EM-38DD vertical dipole conductivity
- EM-38DD horizontal dipole conductivity

The operator can then determine if there are areas in the field that need greater coverage, and can continue the survey from any location in the field.

The operator can save these contour files as grid files, which can be used for additional data analysis back in the laboratory. The grid files can be exported to Excel and can then easily be imported into a GIS system such as ArcView.

EcoTech Research has also created a stand-alone gridding and contouring program that can be used to import raw data or grids that have been created by the ETREMS program. The user interface is very similar to that of the ETREMS program. Grid images can be printed, or they can be screen captured and saved to a file. The various geophysical data grids created during a survey can be manipulated in other programs, such as Excel. For example, the ratio of two grids, such as the EM-38 vertical component to the horizontal component, can be saved as a grid. This grid can then be imported into the gridding and contouring program and displayed as a contour on the screen.

The electronics and computer programs were successfully field tested. A contour map was created for each of the four geophysical data sets provided by the EM-38DD and EM-31 instruments. These were compared to contour maps that had previously been created using only a single instrument.

Conclusions
The Saskatchewan Ministry of Agriculture and Agriculture Canada now have available to them a program that will allow them to record data from three instruments simultaneously.

Effect of instrument height on depth penetration

When an EM-38 or EM-31 is on the ground, each instrument measures a weighted average of the conductivity of a volume beneath the instrument. As the instrument is raised above the ground, this volume changes, and the measured conductivity values also change. By taking several readings at different instrument heights, we will be sampling different volumes of soil. If we have multiple readings at different heights we may be able to determine how conductivity changes with depth. One research group has already built an instrument that can take multiple readings at one point (Figure 12). The disadvantage of this instrument is that the cart must stop at each point as the EM-38 instrument is raised to different elevations.

Figure 12: Mobile salinity assessment vehicle (Rhoades et al, 2006)

The relative response to the primary magnetic field created by the Geonics EM-31 and EM-38 varies with depth. The shape of the relative response curve depends on whether the instrument is in the vertical or horizontal dipole orientation. When the instrument is in the vertical dipole orientation, the relative signal response with depth is:

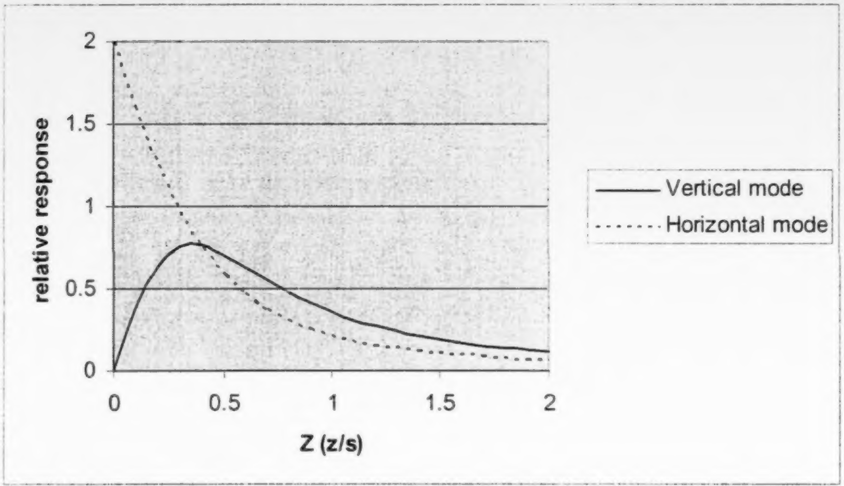
$$S(Z) = 4\frac{Z}{(4Z^2+1)^{(3/2)}} \qquad (1)$$

In the horizontal dipole orientation the relative signal response is:

$$S(Z) = 2 - \frac{4Z}{4Z^2+1} \qquad (2)$$

S(Z) is a function of the normalized depth Z. Z is equal to z/s, where z is the depth from the instrument and s, the intercoil distance, is the distance between the transmitter and receiver. The intercoil distance is 3.66 meters for the EM-31, and 1 meter for the EM-38. Z is measured from the instrument downwards. The relative signal strength with normalized depth is shown in Figure 13.

Figure 13: Relative signal response with depth when the instrument is at the surface



The apparent conductivity measured by the EM-31 and EM-38 instruments is:

$$\sigma_{apparent} = \int_0^\infty \sigma(Z)\, S(Z)\, dZ\,, \qquad (3)$$

$$\sigma_{apparent} = \int_0^H \sigma(Z)\, S(Z)\, dZ + \int_H^\infty \sigma(Z)\, S(Z)\, dZ\,, \qquad (4)$$

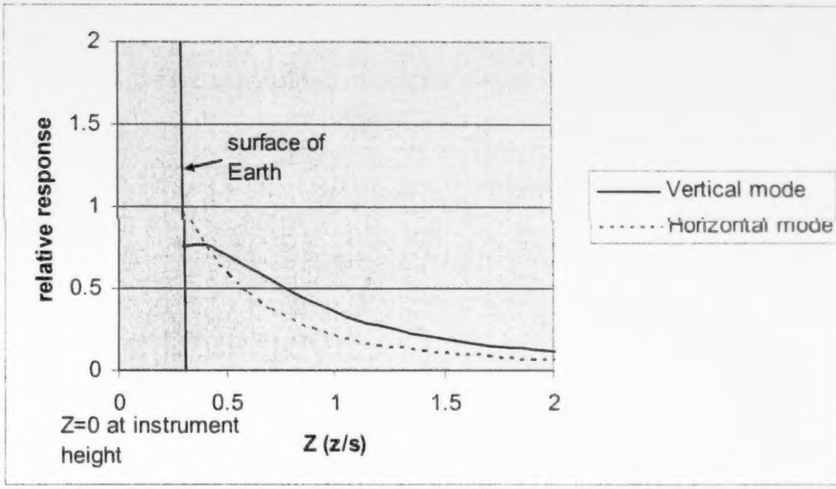
Z=0 occurs at the height of the instrument, σ(Z) is the conductivity of the earth with depth, and H is the height of the instrument in normalized units (H = h/s).

Since the conductivity of air is effectively zero, the first integral vanishes, and the apparent conductivity measured at the instrument is given by:

$$\sigma_{apparent} = \int_H^\infty \sigma(Z)\, S(Z)\, dZ \qquad (5)$$

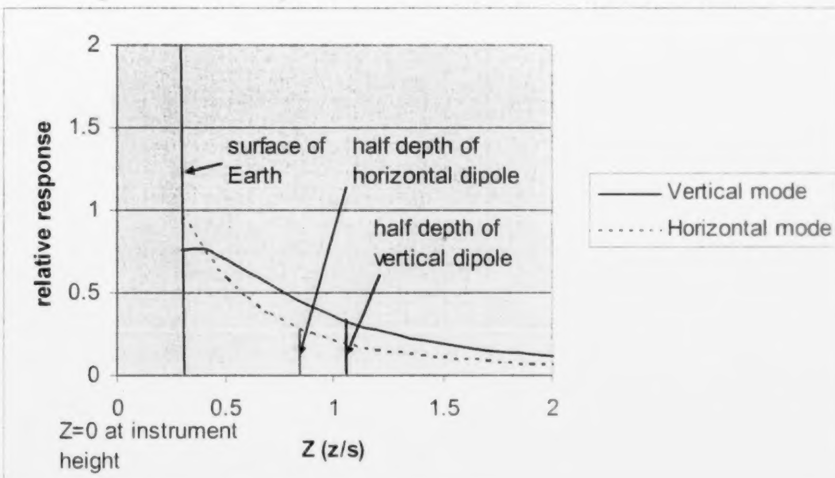
The apparent conductivity is affected only by the portion of the primary field that penetrates beneath the surface. As a consequence, the only part of the relative response curve, S(Z) that contributes to the reading is the portion which is below the surface, as shown in Figure 14.

Figure 14: Effective relative signal response with depth when the instrument is above the surface



Several methods are used to measure the depth of penetration of electromagnetic fields. We will use the half depth as one measure. The half depth represents the depth at which half the relative response of the secondary field is above this depth and half is below (Figure 15). The half depth is often used in planning electrical and electromagnetic surveys, as it allows the researcher to determine the depth above and below which half the current flow or primary electromagnetic field will penetrate.

Figure 15: Half depth when the instrument is above the surface



The half depth is defined by equation 6.

$$\int_{H}^{Halfdepth} S(Z)\,dZ = \int_{Halfdepth}^{\infty} S(Z)\,dZ\,, \qquad (6)$$

Solving this equation for the vertical and horizontal orientations of the Geonics instruments, and transforming the results from normalized units to meters, yields the following half depths:

$$Halfdepth_{vertical} = \frac{1}{2}\sqrt{16\,h^2 + 3\,s^2}\,, \qquad (7)$$

$$Halfdepth_{horizontal} = \frac{3\,s^2 - 8\,h^2 + 4\,h\sqrt{4\,h^2 + s^2}}{8\sqrt{4\,h^2 + s^2} - 16\,h}\,, \qquad (8)$$

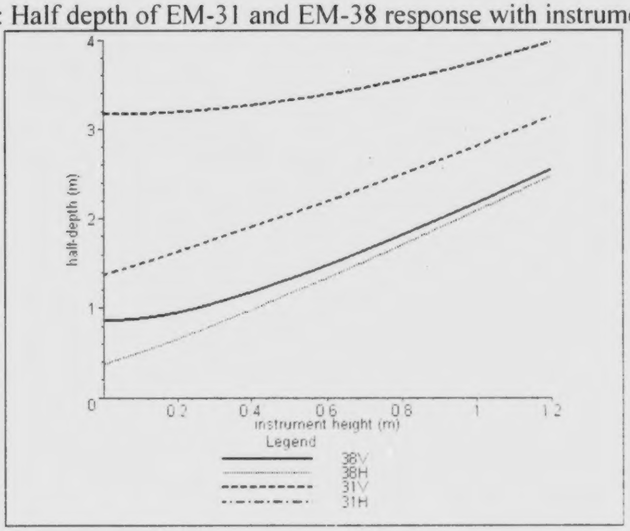
In these formulas h is the height of the instrument in meters, s is the intercoil separation of the Geonics instrument, and Halfdepth is the depth at which half of the relative contribution comes from above this depth and half comes from below. The values h, s and Halfdepth are all measured in metres. When the instruments are at the surface, the half depth of each instrument is shown in Table 1.

Table 1: Instrument half depths at the surface

Instrument	Half depth (m)
EM-31 vertical	3.17
EM-31 horizontal	1.37
EM-38 vertical	0.87
EM-38 horizontal	0.38

As the instruments are lifted off the ground, the half depths increase (Figure 16).

Figure 16: Half depth of EM-31 and EM-38 response with instrument height



Increasing the height of each instrument increases its depth of penetration as measured by the half depth. The depth of penetration increases more quickly with height for the horizontal dipole than for the vertical dipole. This increase in depth of penetration is quite significant in the case of the EM-38. At the surface the depth of penetration of the EM-38 in the vertical dipole orientation is 2.31 times greater than when the instrument is in the horizontal dipole orientation. At a height of 0.5 meters and above the depth of penetration is almost identical for the horizontal and vertical dipole orientations. At this height the vertical dipole component is only 15% greater than horizontal component. At a height of 1 meter the depth of penetration of the vertical dipole field is only 4% greater.

While the depth of penetration increases with instrument height, the apparent conductivity readings measured by the instrument may not increase with height. The actual instrument reading is given by:

$$\sigma_{apparent} = \int_{H}^{Halfdepth} \sigma(Z)\, S(Z)\, dZ + \int_{Halfdepth}^{\infty} \sigma(Z)\, S(Z)\, dZ \quad (11)$$

If the material above the half depth is more conductive than the material below it, then the upper layer contributes more to the apparent conductivity reading than the lower layer. If the material above the half depth has an average conductivity of 500 mS/m and the material below has a conductivity of 1 mS/m, almost the entire signal is due to the material above the half depth. In a case such as this increasing the instrument height will increase the half depth, but the additional low conductivity material now found above the half depth will have a negligible impact on the measured apparent conductivity.

Conclusions

Raising the height of the Geonics instruments above the surface does not decrease the penetration depth of the instrument. It does the opposite. If we have some prior knowledge that a target is at a certain depth, we can set the instrument high enough so that a substantial portion of the effective primary field will reach this depth.

Attaching the EM-38DD to an ATV

Ground conductivity surveys using an EM-38DD usually place the instrument in a sled which is dragged behind a vehicle (Figure 17). Sometimes there is an advantage to being able to adjust the height of the instrument. The survey operator may want a greater depth penetration which can be achieved by increasing the height of the instrument. There may be obstacles in a field through which it may be difficult to pull a sled. In some instances it may be possible to go faster if the EM-38DD instrument is attached to the survey vehicle than if a sled has to be towed behind the vehicle. There are also advantages to using an adjustable boom which can change the height of the instrument depending on field conditions.

Figure 17: Towing the EM-38DD on a sled (PFRA)

If the EM-38 is placed in on adjustable boom, it is possible to make several readings at one location, but at various heights above the ground. This procedure can be carried out at selected areas in a field, such as at the location where soil cores are taken. If we have several apparent conductivity readings at different heights, this will provide us with information about how the soil conductivity varies with depth.

A major concern when attaching a ground conductivity meter close to a vehicle is that the metal and electronics in the vehicle may result in incorrect measurements of the apparent ground conductivity. In order to determine the extent of this interference, we carried out a series of experiments where we made ground conductivity measurements at various heights with and without a vehicle present.

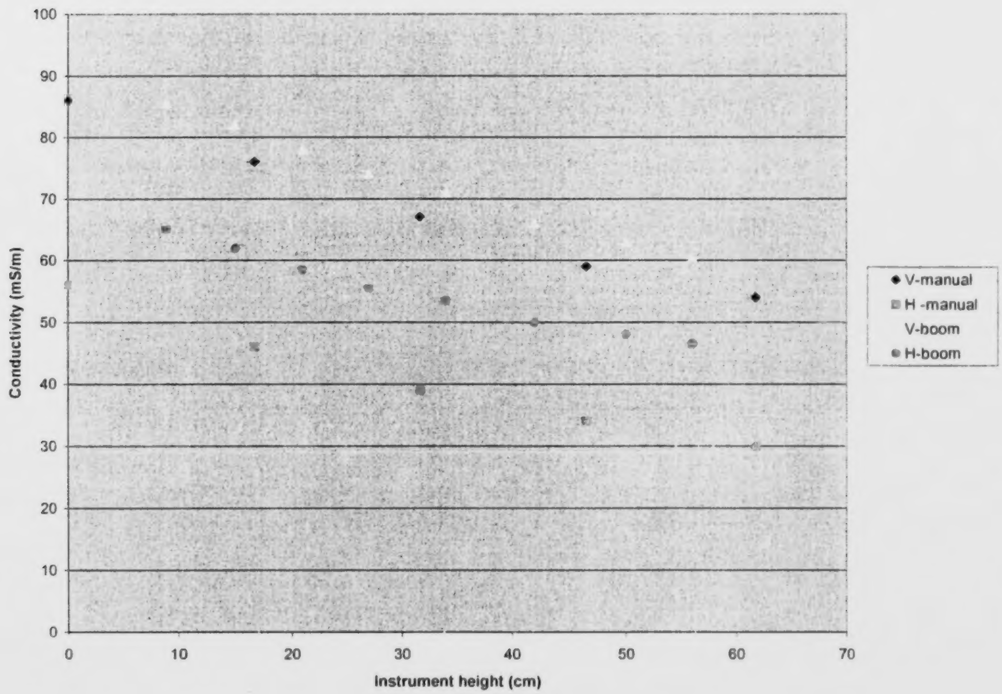
We attached a boom that extended 1.5 meters in front of a 2001 Bombardier Traxter XL, model 7427 all terrain vehicle (Figure 18). The boom was adjustable, and could be quickly raised or lowered with an electric motor. We made measurements at five test sites. The EM-38DD was placed on a non-conducting wooden ladder, and measurements were made from ground level up to 1.5 meters. We then attached the EM-38DD to the boom, and positioned the ATV so that the EM-38DD was in the same location as for the previous readings. Several readings were made at heights up to 0.6 meters.

Figure 18: EM-38DD attached to an adjustable boom (PFRA)

We took measurements using the boom when the ATV motor was running and when it was turned off. We found there was no difference in the measured values at any height. We did notice that the readings changed if the electric motor that moved the boom up and down was turned on. Normally this motor will be turned off during a soil conductivity survey.

When we plotted the results for each site we found that measurements taken in the boom overestimated the apparent conductivity values. This was certainly due to the presence of conductive materials in the ATV. The boom itself was made of non-conductive material. In all cases the magnitude of the conductivity readings decreased with the instrument height (Figure 19, Appendix B).

Figure 19: Site 1 – manual and ATV conductivity measurements



There was a strong linear relationship between the measured apparent conductivity and the instrument height. For the vertical dipole measurements the slopes of the manual and boom readings with height were almost identical.

Site 1

Vertical dipole

Manual	Apparent conductivity = -0.529height + 84.974	$R^2 = 0.989$
ATV	Apparent conductivity = -0.537height + 89.533	$R^2 = 0.993$

If the slopes are identical with instrument height, then the only difference between the readings is a constant value. In the above case the ATV reading will overestimate the correct values by 89.533 – 84.979, or 4.559 mS/m.

The same was also true for the horizontal dipole measurements.

Horizontal dipole

Manual	Apparent conductivity = -0.419height + 54.121	$R^2 = 0.972$
ATV	Apparent conductivity = -0.393height + 67.353	$R^2 = 0.981$

In this case the readings taken when the EM-38DD is on the boom will overestimate the correct value by 13.232 mS/m, regardless of the height of the boom above the ground.

This linear relationship was present at all five test sites. R^2 values ranged from a low of 0.943 to a high of 0.996. The overestimates at each site were similar in size.

Table 2: Conductivity overestimates using the EM-38DD on a boom

	Vertical dipole	**Horizontal dipole**
Site 1	4.559	13.232
Site 2	5.783	13.66
Site 3	7.249	14.963
Site 4	6.369	14.837
Site 5	6.007	14.406
Average	5.993	14.220

Estimates of the apparent conductivity can be made by subtracting 6 mS/m from the vertical dipole measurements made with the boom, and by subtracting 14 mS/m from the horizontal dipole measurements made with the boom.

Conclusions

An EM38-DD can be placed on a mobile boom attached to a vehicle. Corrections can be made to the vertical and horizontal dipole data by subtracting a constant from the measurements. The size of this constant will depend on the vehicle that the EM-38DD is attached to, and on how far the boom extends from the vehicle.

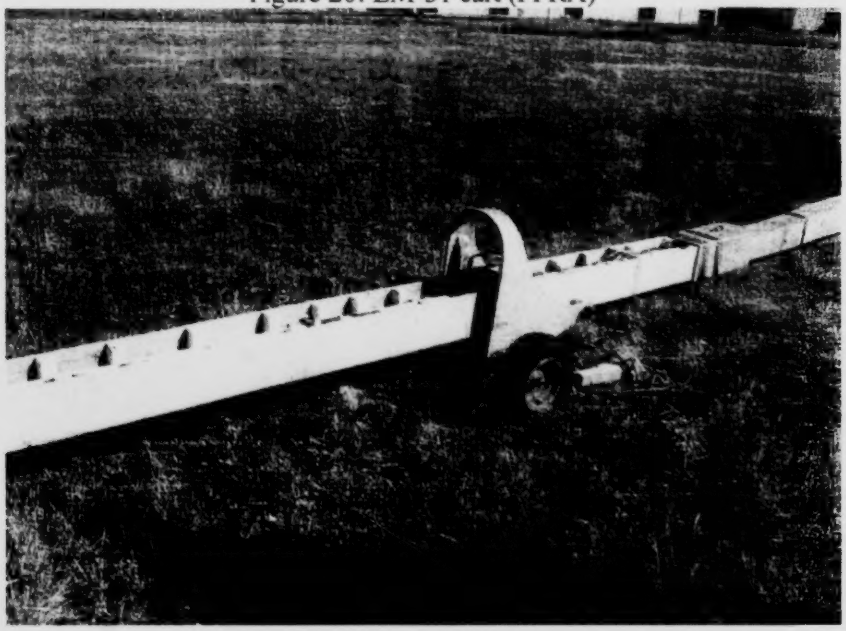
Attaching the EM-38 to a boom requires less time for equipment set up and dismantling than for an EM-38 towed in a sled. In addition the operator can quickly make several readings at various instrument heights at locations of interest within a field. These readings can be used to obtain more information about conductivity variations with depth.

Attaching the EM-31 to an ATV

Most ground conductivity surveys using an EM-31 place the instrument on a sled or in a cart and tow it behind a vehicle. This is done in order to minimize interference from the conductive parts of the vehicle. We wanted to determine if an EM-31 could be attached to a vehicle directly. This provides a major advantage if multiple instruments are used in a survey. If each instrument has to be placed in a separate cart or sled, then we end up with a string of carts being towed by the vehicle. This can become unwieldy as the number of instruments increases. Another advantage of attaching ground conductivity meters to the vehicle is that it decreases the amount of time to set up and take down the equipment. A third advantage is that if we attach the instruments to a vehicle rather than dragging them around, they will be at a higher elevation above the surface. Increasing the height of the ground conductivity meters results in a greater depth penetration.

We wanted to determine how far away the EM-31 would have to be from the side of vehicle before interference became significant. The vehicle we used was a 2001 Bombardier Traxter XL, model 7427 all terrain vehicle. This ATV had been used to pull a cart holding the EM-31. The distance between the cart and the centre of the EM-31 instrument was approximately 8 meters (Figure 20). At this distance the ATV caused little interference with the EM-31 measurements (see Appendix C). The height of the EM-31 was 0.43 meters above the ground.

Figure 20: EM-31 cart (PFRA)



Our interference tests measured the level of interference due to changes in the following parameters:

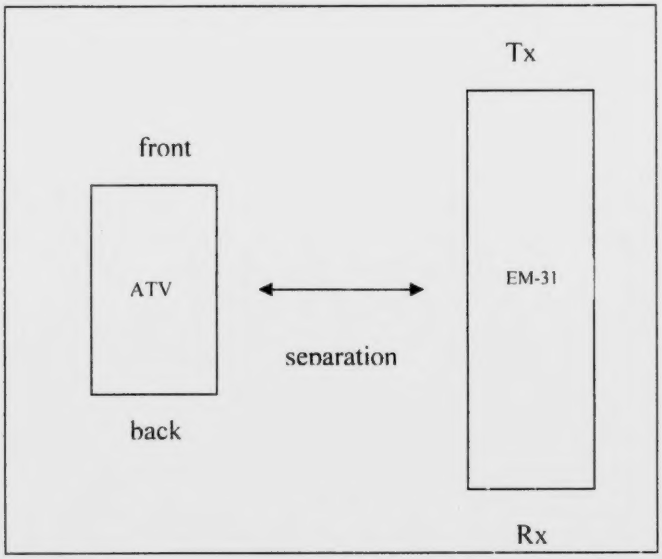
- The height of the EM-31 above the surface.
- The orientation of the EM-31 (transmitter to the front or behind the ATV).
- The separation of the EM-31 and the ATV.
- The offset of the centre of the EM-31 from the centre of the ATV.

We selected a test location and measured the EM-31 conductivity readings at 4 heights above the surface (0.62m, 0.76m, 0.92m, and 1.07m). The measurements were made by placing the EM-31 on a non-conducting wooden ladder while the ATV was some distance away. To test the effect of interference we moved the ATV parallel to the EM-31. Tests were carried out at different separations between the ATV and the EM-31. In each case the centre of the ATV was placed adjacent to the centre of the EM-31. Measurements were again taken at each height as before. Measurements were taken twice at each elevation, once with the EM-31 receiver facing forward, and once with the EM-31 receiver facing the back of the ATV. We also moved the ATV forward to determine the affect of such a displacement on the level of interference.

We first tested PFRA's current instrument setup (Figure 20) by measuring the conductivity as the EM-31 was moved behind the ATV (Appendix C). The EM-31 was oriented so that the receiver coil was closest to the ATV. The measurement errors dropped to about 2% once the centre of the EM-31 was 4 meters behind the ATV. This distance could probably be shortened if the EM-31 transmitter was closest to the ATV.

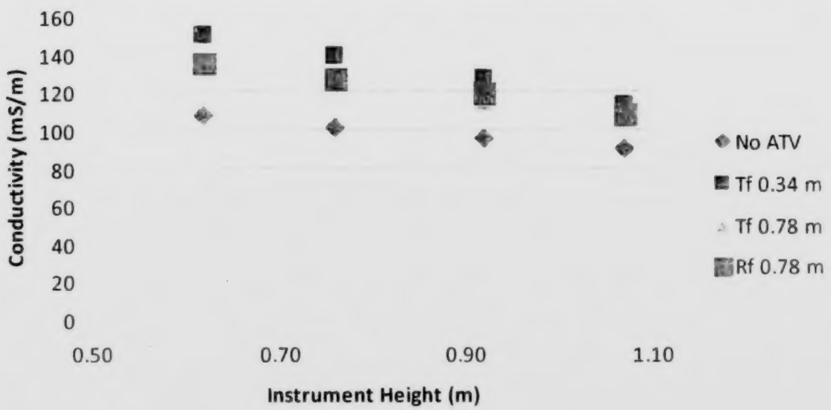
Case 1: The centre of the ATV is aligned with the centre of the EM-31

Figure 21: ATV and EM-31 alignment for Case 1



In this case the conductivity measured by the EM-31 in the presence of the ATV is always larger than the conductivity measured when the ATV was absent (Figure 22, Appendix C). In this chart the abbreviations Tf and Rf stand for transmitter to the front and receiver to the front. The numbers to the right of these abbreviations are the distance between EM-31 and the edge of the ATV.

Figure 22: The centre of the ATV is parallel to the centre of the EM-31



The presence of the ATV results in an overestimate of the ground conductivity. Errors ranged from 40% to 13% depending on the height, separation and orientation of the EM-31 instrument. Increasing the separation between the ATV and the EM-31 reduces the measurement error. Switching the orientation of the EM-31 with respect to the EM-38 has only a marginal effect on reducing the measurement error.

We conducted a regression analysis to determine if we could predict the actual conductivity values from the measured conductivity values at each height. For each case there was an excellent linear relationship between the predicted and actual conductivities. The regression equations are:

Tf 0.34 m Predicted Conductivity = 0.461 * Measured Conductivity + 37.3 (R^2=0.991)

Tf 0.78 m Predicted Conductivity = 1.000 * Measured Conductivity + 28.0 (R^2=0.996)

Rf 0.78 m Predicted Conductivity = 0.634 * Measured Conductivity + 20.8 (R^2=0.996)

The actual and predicted results were:

Table 3: Predicted conductivity values for EM-31 instrument adjacent to an ATV

Height	No ATV	Predicted Tf 0.34 m	Predicted Tf 0.78 m	Predicted Rf 0.78 m
0.62	107	106	107	106
0.76	101	101	101	100
0.92	95	96	95	94
1.07	90	89	90	87

Case 2: The centre of the EM-31 is displaced from the centre of the ATV; ΔX = 0.34m

In this set of experiments the EM-31 was placed parallel to the ATV with a separation of 0.34 meters. The distance between the centres of the EM-31 and the ATV were varied by moving the ATV forwards. The orientation of the EM-31 was also varied, so that in some cases the transmitter was toward the front of the ATV (Tf), while in other cases the receiver was toward the front of the ATV (Rf). Measurements were taken at different instrument heights. The values of Y represent how far the centre of the ATV was moved in front of the centre of the EM-31 (Figure 23).

Figure 23: ATV and EM-31 alignment for Case 2

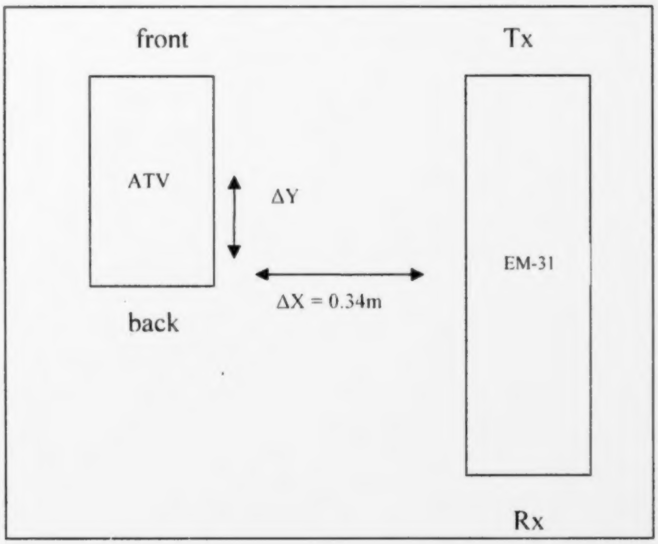
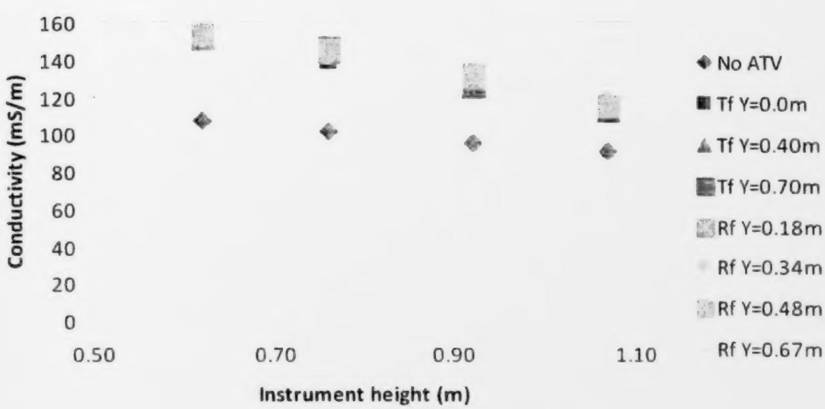


Figure 24: The EM-31 is 0.34 meters to the side of the ATV

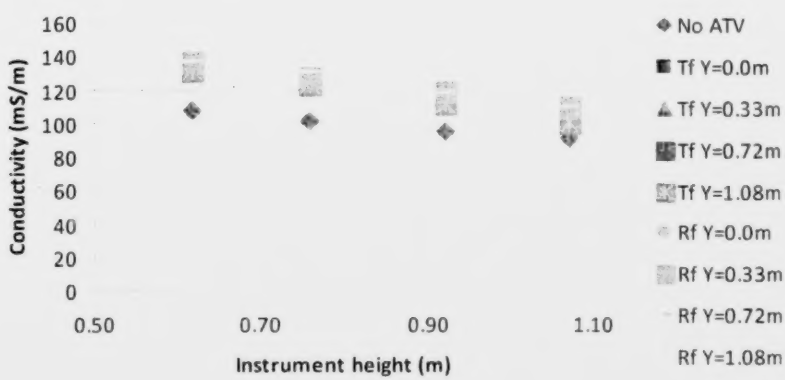


Measurements were taken at each height for seven different combinations of EM-31 orientation and offset of the ATV from the EM-31. In all cases the EM-31 overestimates the apparent conductivity measurements when the ATV is present (Figure 24, Appendix C).

Case 3: The centre of the EM-31 is displaced from the centre of the ATV; ΔX = 0.78m

The instrument configuration for this set of experiments is identical to that of Case 2, with the sole exception that the separation between the two instruments, ΔX, is now 0.78 meters. When the EM-31 is attached to the ATV it overestimates the apparent ground conductivity (Figure 25, Appendix C). The size of the error decreases with the height of the instrument about the ground. The error is smallest (11% error) in the transmitter forward (Tf) orientation with the ATV 1.08 meters ahead of the EM-31. It appears that the EM-31 is most sensitive to interference from the ATV when the receiver is closer than the transmitter to the ATV. Again there is a strong linear relationship between the measurements taken by the EM-31 in the presence and absence of the ATV.

Figure 25: The EM-31 is 0.78 meters to the side of the ATV



Conclusions

In all cases the EM-31 overestimates the apparent conductivity values if an ATV is nearby. The higher the instrument is positioned, the lower the error. As the separation between the ATV and the EM-31 increases, the magnitude of the error decreases. The error is smallest when the EM-31 is in the transmitter forward (Tf) position. Changing the ΔY displacement between the centres of the ATV and EM-31 also affects measurement accuracy. If the transmitter is forward, then increasing ΔY will decrease the error. If the receiver is forward, increasing ΔY will either have no effect or will increase the error.

Future work

The results so far are encouraging, and it is likely that future soil surveys can be carried out by hanging a boom over the side of the ATV. The EM-31 can be attached to this

boom in the transmitter forward orientation, and moving the centre of the EM-31 behind the centre of the ATV.
An experiment can be carried out to determine how long the boom should be in order to reduce the measurement errors to a tolerable magnitude.

It may also be possible to place the EM-31 close to the ATV (less than 1 meter) and make corrections to the measured data. This was successfully done when the EM-38 was placed in a boom at the front of the ATV. A set of experiments can be carried out that will measure the EM-31 readings with and without the ATV at a number of test sites. It may be possible to calculate a simple correction factor that will be valid for the instrument height and orientation.

Improving resolution when EM measurements are made above ground

Mapping spatial changes in conductivity

Increasing the height of the EM-38DD above the surface lowers the amplitude of the apparent conductivity reading. This will affect the ability of a survey operator to distinguish lateral changes in conductivity in a field. In order to determine the effect of instrument height on mapping conductivity variations we created three different soil conductivity models:

Model 1	The soil consists of a single layer containing lateral variations in conductivity (Figure 26).
Model 2	The soil consists of two layers. There are horizontal variations in the top layer while the bottom layer has constant conductivity (Figure 27).
Model 3	The soil consists of two layers. There are horizontal variations in the bottom layer, while the top layer has constant conductivity (Figure 28).

Figure 26: Model 1: Single layer

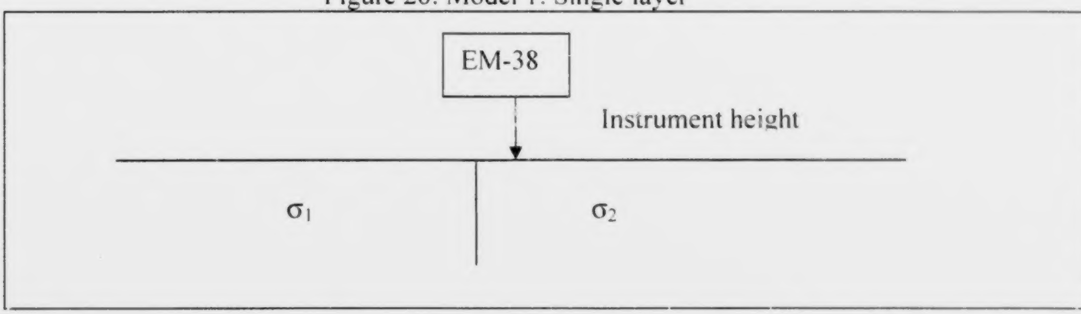


Figure 27: Model 2: Two layers– conductivity changes in layer 1

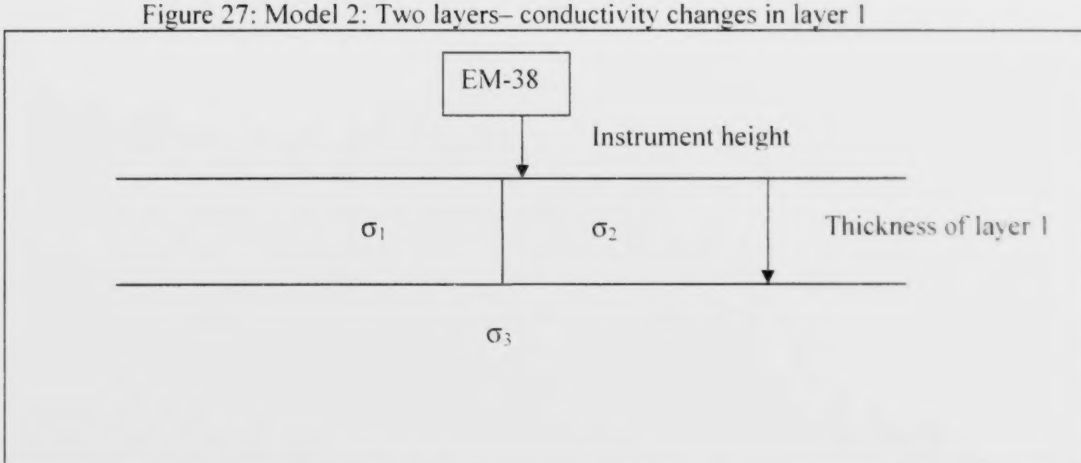
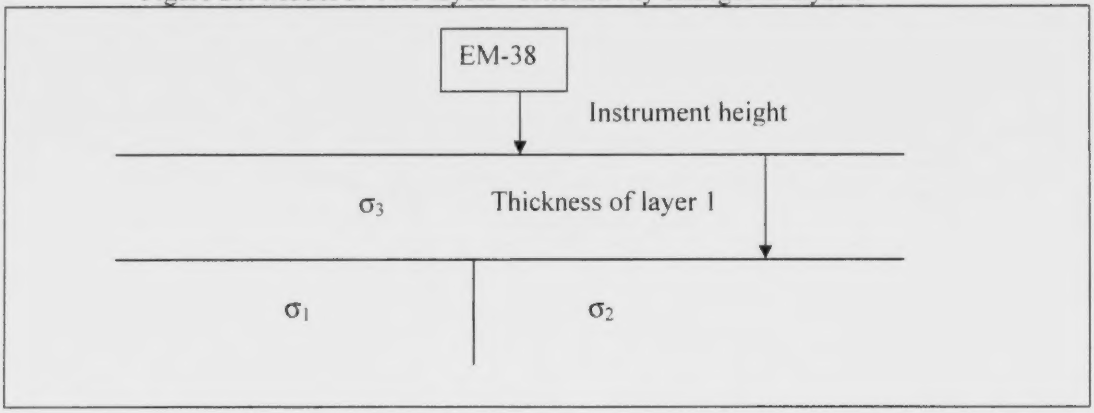


Figure 28: Model 3: Two layers– conductivity changes in layer 2



Horizontal changes in conductivity were measured in two ways:

- the difference in the apparent conductivity between the left and right hand side at each instrument height:

$$\text{Difference} = \sigma_{apparent,left} - \sigma_{apparent,right} \quad (9)$$

- the ratio of the left hand conductivity readings to the right hand conductivity readings at each instrument height:

$$\text{Ratio} = \sigma_{apparent,left} / \sigma_{apparent,right} \quad (10)$$

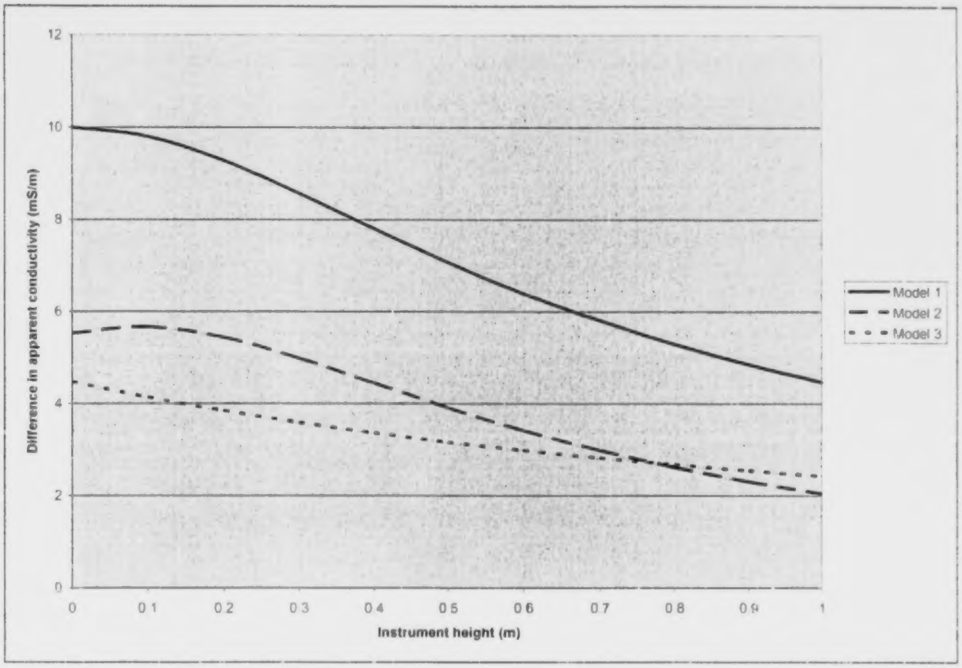
The apparent conductivity differences and ratios for all three models were calculated using numerous combinations of conductivity values for σ_1, σ_2, σ_3 and the thickness of layer 1. The effectiveness of the two methods to discriminate changes in soil conductivity was compared. Both measures were also tested using field data.

Mapping spatial changes in conductivity – modeling results

The changes in the difference in conductivity readings between the left and right hand sides of each model are displayed in Figures 29 and 30. These figures are based on the following parameters: σ_1 = 100 mS/m, σ_2 = 90 mS/m, σ_3 = 50 mS/m, and thickness of the top layer = 1 m.

Figure 29 displays the drop in differences with height for the EM-38 in the vertical dipole position. For a one layer case the difference in apparent conductivity drops rapidly after a height of 0.2 meters. For Model 2, the differences increase up to 0.1 meter in height, and then drop rapidly. For Model 3 the differences in conductivities drop slowly with instrument height. Differences in apparent conductivity for the EM-31 instrument decrease more slowly with increased instrument height than they do for the EM-38.

Figure 29: Differences in apparent conductivity with instrument height - EM-38 vertical dipole



In the horizontal dipole orientation the difference in apparent conductivity between the two soil profiles drops rapidly with height for models 1 and 2. This drop is much larger than for the vertical dipole orientation. For model 3, the drop is similar in magnitude to that when the instrument is in the vertical dipole orientation.

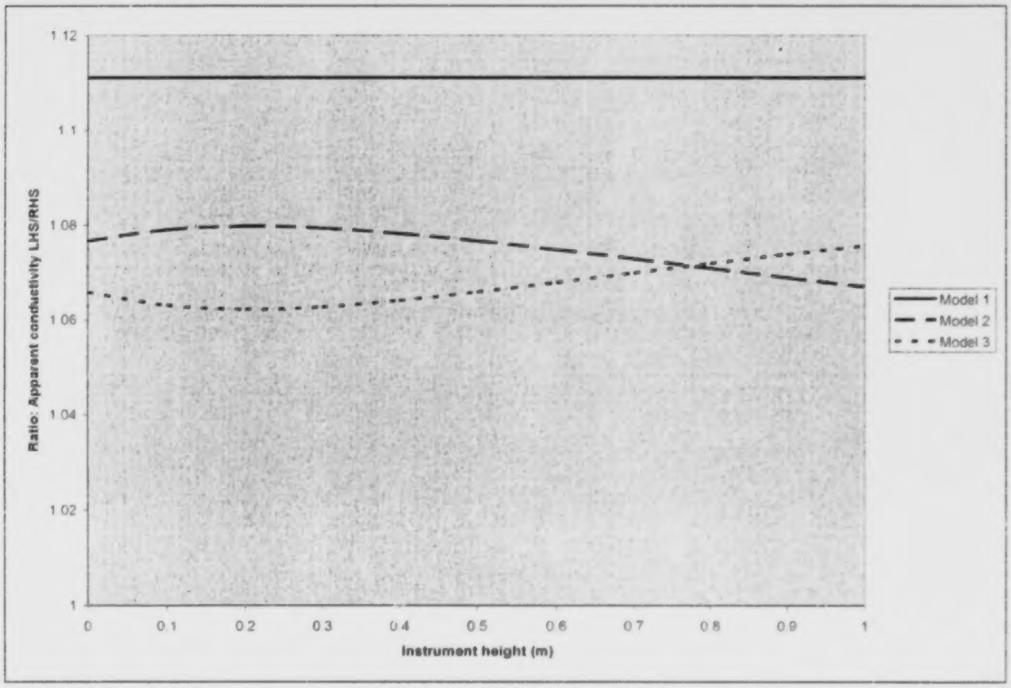
As the instrument height increases, the ability of the EM-38 to detect lateral differences in apparent conductivities decreases. When the EM-38 is in the vertical dipole orientation, the decrease in the difference between conductivities is fairly small up to 0.5 meters. When the instrument is in the horizontal dipole orientation, the decrease in the difference in conductivities is substantial at a height of 0.5 meters. When the EM-31 instrument is used, the differences drop more slowly with height than for the EM-38 instrument.

The second measure used to detect horizontal changes in conductivity was the ratio of the left hand conductivity reading to the right hand conductivity reading at each height. A ratio of 1 indicates that the apparent conductivities are identical, while a ratio greater than 1 indicates that the apparent conductivity is higher on the left hand side.

For the single layer model the ratio of apparent conductivity readings over the left and right layers remains the same regardless of instrument height (Figure 30). For models 2 and 3 the ratio of readings made in the vertical dipole orientation are fairly constant up to

an instrument height of 1 meter. If we compare Figure 29 with Figure 30 we see that the ratio measure drops far more slowly with instrument height than the difference measure.

Figure 30: Ratio of apparent conductivities (LHS/RHS) - EM-38 vertical dipole



When the readings are made in the horizontal dipole orientation, the ratios drop with height when conductivity changes are present in the top layer (Model 2). The ratios increase with height when conductivity changes are present in the second layer (Model 3).

The extent to which the differences and ratios drop when the EM-38 is placed at a height of 1 meter above the ground is displayed in the following table. For model 1 the conductivity difference between the left and right hand sides at a height of 1 meter are only 45% of what they would be if measured at the surface. The ratio of the left and right hand side is identical at a height of 1 meter to that measured at the surface.

Table 4: Change in differences and ratios for each model when measurements are taken at a height of 1 meter: EM-38

	Vertical Dipole			Horizontal Dipole		
	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3
Difference	45%	37%	54%	24%	15%	52%
Ratio	100%	99%	101%	100%	98%	103%

Figures 29 and 30 and Table 4 indicate that the ratio measure will be superior in detecting conductivity changes when the differences are small to begin with, and become smaller when the instrument is situated above the surface. This is true for all three models in both the horizontal and vertical dipole instrument orientations.

In the above examples the ratio of apparent conductivities is less sensitive to instrument height than the differences in conductivity readings. When EM-31 surveys are made, both the differences and ratios change less with increasing instrument height than they do with an EM-38.

Mapping spatial changes in conductivity – field results

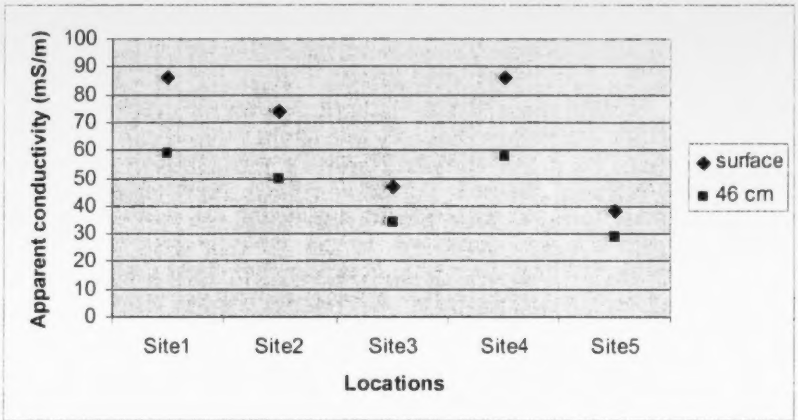
Field measurements were carried out to test each method of determining conductivity differences. Readings were taken at 5 sites in a field with the EM-38 in both the vertical and horizontal orientations, and at three instrument heights. The apparent conductivities at each site are shown in Figure 31. As the instrument height increases, the apparent conductivities at each site decrease and the differences between the readings also decrease. The differences in measurements between sites 1 and 2 are shown below.

Table 5: Difference in EM-38 readings between sites 1 and 2

Height (m)	Vertical Dipole Difference (mS/m)	Horizontal Dipole Difference (mS/m)
0.00	12	10
0.46	9	6

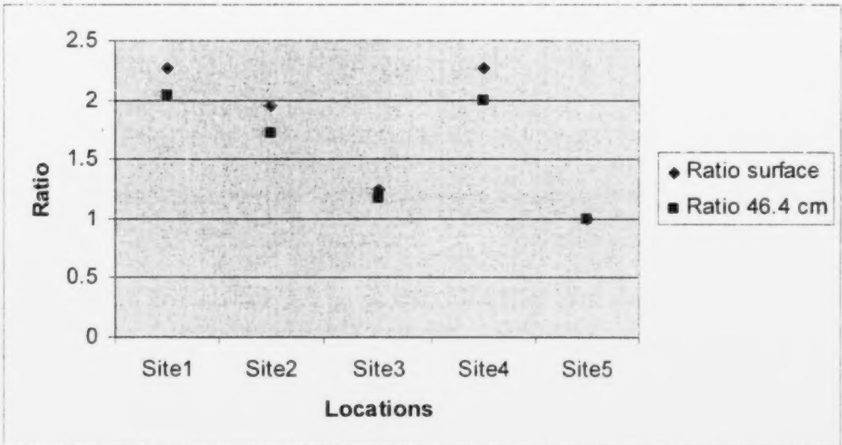
The increase in sensor height has reduced the size of the difference in apparent conductivity readings between the two sites. The implication for soil surveys is that it will be harder to distinguish conductivity differences using this method as the instrument height increases. This result will be particularly true if the differences in conductivities between the two soil units are small. In such cases raising the instrument may result in differences that are indistinguishable from noise. In saline fields the differences in conductivity between saline and non-saline areas is so large that each area can easily be identified even if the instrument is situated high above the ground. In non-saline areas the conductivity difference between soils will be much smaller, and using conductivity differences to locate different soils may be affected by instrument height.

Figure 31: Apparent conductivity readings with instrument height (vertical dipole)



The second method of distinguishing conductivity differences by using conductivity ratios is far less sensitive to instrument height. Apparent conductivity ratios were calculated by dividing the readings for sites 1 to 4 by the apparent conductivity measured at site 5. Raising the instrument to a height of 46 centimetres had little effect on the ratios (Figure 32). Even at a height of 150 centimetres the differences in ratios between sites were easily discernable.

Figure 32: Ratio of apparent conductivities with instrument height (vertical dipole)



Conclusions

If the purpose of a conductivity survey is to locate areas of varying conductivity over a field, then raising the instrument height will make it harder to distinguish different conductivity areas using traditional methods. However, if lateral changes in apparent conductivity are measured using conductivity ratio maps, there is almost no loss of information compared to when measurements are made at the surface. Raising the instruments will have little effect on the survey operator's e ability to distinguish different soils.

Analysis of existing height correction formulas - EM-38DD

Soil survey operators who have taken above ground measurements sometimes want to correct their readings for the instrument height. They want to estimate what the readings would have been if the instrument had been at the surface. One reason for this is that it is difficult to compare the results of surveys that were made at different instrument heights. Also some surveyors have tried to determine soil conditions, such as soil type and salinity, from the amplitudes of the conductivity measurements.

If it known that the soil conductivity is uniform down to the effective depth of penetration of the instrument, then a simple correction can be used. One researcher (Korsaeth, 2006) recently developed the following height correction formulas for the EM-38, where h is the instrument height above the ground in meters:

$$EMv\_corr := EMv\sqrt{4\,h^2+1} \qquad (11)$$

$$EMh\_corr := \frac{EMh}{-2\,h+\sqrt{4\,h^2+1}} \qquad (12)$$

Korsaeth field tested these correction formulas at heights of 20, 40 and 60 cm for the EM38 in both the horizontal and vertical dipole orientations. His field tests showed that while the correction formulas gave a more accurate estimate of the ground level readings than the readings made at height, there was still a systematic deviation from the ground level reading. As the instrument height increased, the errors due to the correction formulas also increased.

The derivation of the correction formulas is accurate as long as the assumption that conductivity does not change with depth is true. We tested the accuracy of these formulas for a 2-layered earth. In this case the apparent conductivity measured above the surface of the earth is a function of four variables: the conductivity of each of the two layers, the depth from the surface to the bottom of the first layer, and the height of the instrument above the surface. We created 10,000 random values for each of these variables. The values of the conductivity were chosen to be between 1 and 1000 mS/m. The values of the depth to the second layer were chosen to be between 0.2 and 15 meters. Error analysis was carried out for instruments at heights of 0.1, 0.2, 0.3, 0.4, 0.5 and 1.0 meters.

The following steps were carried out for each of the 10,000 points:

1. Use a two-layer model to calculate the vertical and horizontal apparent conductivity readings if the instrument was at the surface (height = 0).
2. Determine what the vertical and horizontal apparent conductivity readings would be at the randomly selected instrument height.
3. Apply correction formulas to the apparent horizontal and vertical conductivity values calculated in step 2.
4. Calculate the percentage error for each point by comparing the corrected value determined in step 3 with the surface value determined in step 1:
   (Corrected value – True surface value)/True surface value * 100%

The errors were then analyzed in order to determine the accuracy of the correction formulas under various two-layer conductivity structures. The average absolute error was calculated for all the 10,000 predicted values at each instrument height (Table 6). At all heights the average error was higher for the instrument in the horizontal dipole position. This is consistent with the errors Korsaeth observed when instruments were placed between 0.4 and 1 meter above the surface. The average error increased with the height of the instrument.

Table 6 – Average absolute correction error (%)

Height (m)	EM38 vertical	EM 38 horizontal
1.0	9.30%	21.4%
0.5	3.40%	11.0%
0.4	2.52%	8.55%
0.3	1.86%	6.25%
0.2	1.45%	4.02%
0.1	1.00%	1.86%

Errors are smallest when the conductivity levels in the two layers are roughly the same. Negative errors, where the corrected value underestimates the actual surface value, generally occur when there is a thin top layer overlaying a significantly more conductive second layer. Positive errors, where the corrected value is greater than the surface value, generally occur when a thin conductive top layer overlays a second layer with much lower conductivity. Large conductivity differences between the layers are less likely to lead to errors if the thickness of the top layer is greater than the intercoil distance.

We also find that the proportion of errors that are less than 1% or less than 5% of the true surface value decreases rapidly with instrument height (Tables 7 and 8). At a height of 0.5 meters approximately 30% of all vertical dipole corrections will be accurate to within 1% of the true value and 82% will be accurate to within 5%. This implies that 18% of all corrections will have an error that is greater than 5% of the true value.

Table 7: Proportion of EM-38 vertical dipole calculations accurate to within 1% and 5% of the true surface value

Instrument height (m)	Accurate to within 1%	Accurate to within 5%
1.0	.13	.45
0.5	.30	.82
0.4	.43	.87
0.3	.63	.91
0.2	.74	.92
0.1	.77	.96

When the instrument is in the horizontal dipole position, the proportion of corrections that are accurate becomes quite small as soon as the instrument is raised above 10 centimetres. At a height of 0.2 meters twenty-three percent of all corrections will be in error by more than 5%.

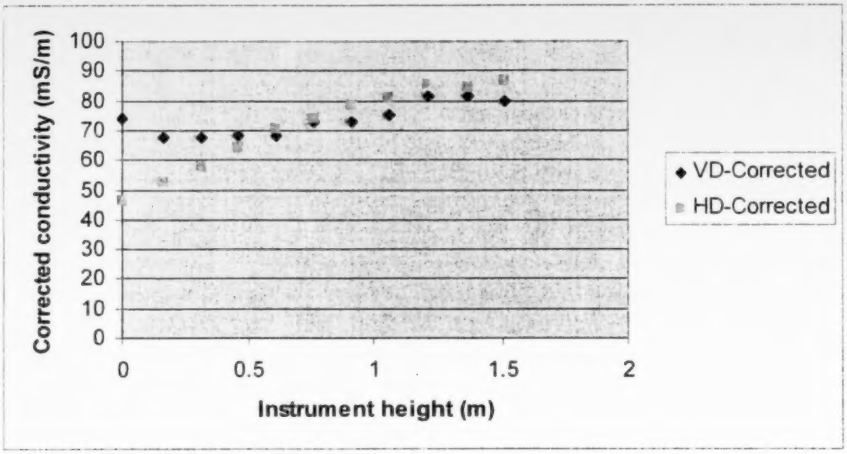
Table 8: Proportion of EM-38 horizontal dipole calculations accurate to within 1% and 5% of the true surface value

Instrument height (m)	Accurate to within 1%	Accurate to within 5%
1.0	.05	.24
0.5	.11	.43
0.4	.13	.52
0.3	.17	.64
0.2	.25	.77
0.1	.49	.91

The accuracy of the corrections can be checked by taking an instrument reading at the regular sensor height and at the surface. The correction formula is applied to the above-surface reading, and is compared with the actual value at the surface. If the error is less than an acceptable value, such as 5% error, then the correction formula can be used. If different soil structures exist in different parts of the survey error, this test can be done for each area. Some areas may have variable soil structures, and the correction factors may work in some areas, but not in others. Major differences in soil properties can usually be determined from a quick display of the conductivity data once the field has been surveyed.

Tests were carried out at five sites in order to determine how well the conductivity correction equations worked. At each site 11 measurements were taken at instrument heights between 0 and 1.5 meters above the ground. Corrections were carried for at each instrument height. Figure 33 displays the corrections at one of the test sites. The value for the vertical and horizontal readings at height = 0 is the actual value measured by the instrument at ground level. The vertical conductivity corrections gave an error of between -9% and +10% of the actual ground reading. The percent error in the horizontal corrections increased from 14% at a height of .165 meters to 89% at 1.513 meters. A similar pattern was present at all five test sites (Appendix D). This indicates that the conductivity is not uniform with depth. If the conductivity were uniform to the effective penetration depth of the instrument, then the ratio of the vertical to horizontal conductivity reading at ground level would be 1.0. At this test site the actual ratio is 1.6, indicating large continuity changes with depth. Under such conditions the correction formulas are likely to result in large estimation errors.

Figure 33: Corrected EM-38 conductivity values by instrument height



If the correction errors are too large, an alternative correction method is possible. Assume that a display of the conductivity readings indicates that there are three major soil structures or units present in a field. Find a representative location within each unit, and measure the apparent conductivity at the usual instrument height and at ground level. For each location calculate the value of CorrectionFactor = Surface Conductivity reading/Height conductivity reading. Multiply all readings within each area by the Correction Factor. This method can be used as an alternative to using the correction equations.

Conclusions

The height correction formulas for the EM-38DD will provide accurate results if conductivity does not change appreciably with depth. If there are two or more conductivity layers within the penetration range of the EM-38DD, then using the formulas derived by Korsaeth can lead to substantial prediction errors. Prediction errors increase with the instrument height, and are higher for the readings made with the horizontal dipole instrument.

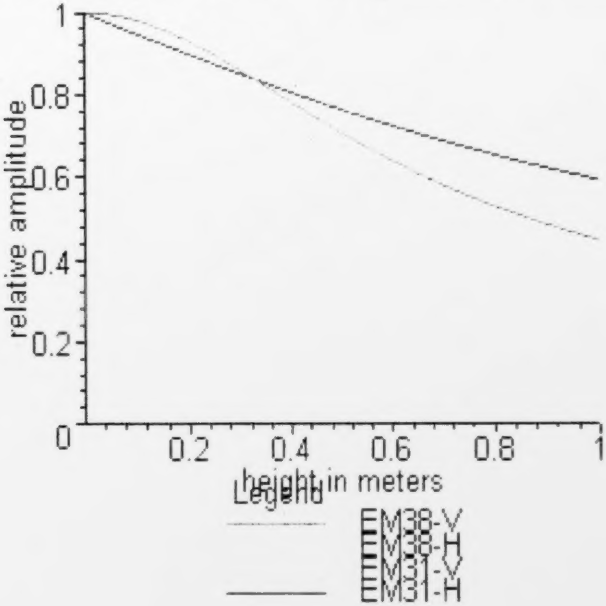
Though the accuracies involved with the correction formulas are limited, there may be situations where having the instruments above the surface may not be a problem for interpreting subsurface data. These correction formulas only tell us what the apparent conductivity is at the surface. Whether an instrument is at the surface gives us no more information about subsurface conditions than if the instrument is above the surface. For a one layer case, the horizontal and vertical surface readings will be identical, and will be the value of the subsurface conductivity. For a one-layer case the height corrected values will be identical to the surface value, and it will also accurately measure subsurface values. For the case where two layers with different conductivities are present, neither the surface readings nor readings at height will tell us anything more about the conductivities in each layer, or the depth to the second layer. Additional information is required in order to solve for these values.

Height correction formulas for the EM31

Conductivity surveys using the EM31 instrument usually place the instrument on a wheeled cart and tow it behind a vehicle. The length of the instrument (3.66 m) makes it difficult to drag in a sled. EM-31 instruments are usually positioned between 10 and 50 cm above the surface. We analyzed what the effect the EM-31 instrument height would have on the apparent conductivity measurements, and created correction formulas for the instrument.

For a single layer earth, the apparent conductivity reading would drop as shown in Figure 34. For both the EM31 and EM38 instruments, the drop in amplitude with height is greater when the instrument is in the horizontal dipole orientation. The drop in amplitude with height also is far greater for the EM38 than the EM31 instrument. This is due to the smaller intercoil distance of the EM38.

Figure 34: Relative drop in EM readings with instrument height



From the above chart we can see that the vertical component of the EM-31 drops quite slowly with increasing instrument height. At a height of 0.3 meters the vertical reading will be 99% of the surface value: at 0.4 meters it will be 98%: at 0.5 meters it will be 96%: at 1 meter it will drop to 88% of the surface value. Most EM-31 instrument surveys are carried out in the vertical dipole position, and are positioned between 0.3 and 0.4 meters above the ground. The instrument used by PFRA in the Regina lab is 0.43 meters above the ground. In such cases the uncorrected values will be accurate to within 2% of the ground reading. When the instrument is placed higher, or if the instrument is in the horizontal position, then height corrections may be made.

We extended Korsaeth's height correction method for the EM-38 to the EM-31 instrument.

The apparent conductivity measured by a vertical and horizontal dipole instruments is:

$$EC_{apparentV\_height} := \frac{\sigma}{\sqrt{4H^2+1}}, \quad (13)$$

$$EC_{apparentH\_height} := \sigma\left(2H + \sqrt{4H^2+1}\right), \quad (14)$$

H is the height of the instrument, measured in normalized units (H = h/s), and σ is the conductivity of the material below the instrument. The height of the instrument in meters is h, and s is the intercoil distance between the transmitter and receiver. For the EM-38 the intercoil distance is 1 meter, and H=h in this case. The intercoil distance of the EM-31 is 3.66 meters, and H will be h/3.66. The resulting correction formulas for the EM31 instrument are:

$$EMv\_corr := EMv\sqrt{.3h^2+1} \quad (15)$$

$$EMh\_corr := \frac{EMh}{-.546h+\sqrt{.3h^2+1}} \quad (16)$$

We tested the accuracy of these formulas when the assumption of a constant conductivity with depth was not true. We created 10,000 random values for each of these variables. The values of the conductivity were chosen to be between 1 and 1000 mS/m. The values of the depth to the second layer were chosen to be between 0.2 and 15 meters. Error analysis was carried out for instruments at heights of 0.1, 0.2, 0.3, 0.4, 0.5 and 1.0 meters.

The following steps were carried out for each of the 10,000 points:

1. Use a two-layer model to calculate the vertical and horizontal apparent conductivity readings if the instrument was at the surface (height = 0).
2. Determine what the vertical and horizontal apparent conductivity readings would be at the randomly selected instrument height.
3. Apply correction formulas to the apparent horizontal and vertical conductivity values calculated in step 2.
4. Calculate the percentage error for each point by comparing the corrected value determined in step 3 with the surface value determined in step 1:

   (Corrected value – True surface value)/True surface value * 100%

The average percent error was calculated for the EM-31 in both the vertical and horizontal dipole orientations for instrument heights ranging from 0.1 to 1.0 meters (Table 9).

Table 9 – Average absolute correction error (%)

Height (m)	**EM31 vertical**	**EM31 horizontal**
1.0	2.05%	5.55%
0.5	1.40%	2.57%
0.4	1.18%	2.03%
0.3	1.02%	1.49%
0.2	0.77%	0.97%
0.1	0.42%	0.47%

The average errors are much less than for the EM-38DD. Even at a height of 1 meter the average error is fairly small.

We also found that the proportion of errors that are less than 1% or less than 5% of the true surface value decreases slowly with instrument height (Tables 10 and 11). At a height of 0.5 meters approximately 73% of all vertical dipole corrections will be accurate to within 1% of the true value and 93% will be accurate to within 5%. This implies that 7% of all corrections will have an error that is greater than 5% of the true value. If the EM-31 is placed in the horizontal dipole position, then 39% of all corrected readings will be accurate to within 1% of the true value, and 87% will be accurate to within 5% of the actual ground value.

Table 10: Proportion of EM31 vertical dipole calculations accurate to within 1% and 5% of the true surface value

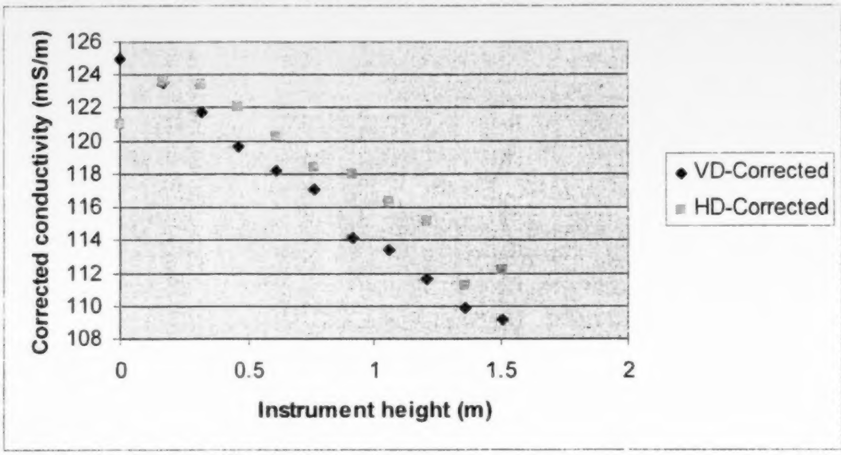
Instrument height (m)	Accurate to within 1%	Accurate to within 5%
1.0	.68	.91
0.5	.74	.93
0.4	.75	.95
0.3	.77	.96
0.2	.81	.98
0.1	.89	.99

Table 11: Proportion of EM31 horizontal dipole calculations accurate to within 1% and 5% of the true surface value

Instrument height (m)	Accurate to within 1%	Accurate to within 5%
1.0	.19	.68
0.5	.39	.87
0.4	.47	.90
0.3	.59	.94
0.2	.73	.97
0.1	.88	1.00

Field tests of the correction equations were carried out at two test sites (Figure 35).

Figure 35: Corrected EM-31 conductivity values by instrument height



At this test site the corrected values of in the vertical dipole position were accurate to within 5% of ground value up to an instrument height of 0.62 meters (Appendix E). At a height of 1 meter the measurements underestimated the true ground values by 9%. The horizontal corrections were accurate to within 5% up to an instrument height of 1.2 meters.

Conclusions

Most EM-31 surveys are carried out with the instrument in the vertical position. If the instrument is less than 0.4 meters above the surface, the conductivity measurements will underestimate surface readings by about 2%. In such cases no corrections need to be made. If the instrument is set higher than this, or if the instrument is in the horizontal orientation, then corrections may be made.

The correction formulas will be accurate if the conductivity is fairly uniform down to the penetration depth of the instrument. Under or overestimates of the ground conductivity values will occur if the conductivity varies with depth. If conductivity does vary substantially with depth, then neither a ground level measurement nor a measurement at any height will tell us how the conductivity values vary with depth. Other methods are needed to measure conductivity changes with depth.

Estimating the conductivities of a two-layered soil

A quick test is available to determine if soil conductivity is uniform down to the effective penetration depth of a ground conductivity meter. A horizontal and vertical dipole reading can be made on the surface. If the two measurements are almost identical we can safely assume that conductivity is uniform, and that a single conductivity layer lies beneath the instrument. In such a case the formulas that make corrections for the instrument height will give accurate results. If the vertical and horizontal readings differ by more than 5%, then the conductivity varies with depth.

In situations where the conductivity is not uniform with depth, the conductivity variations can often be accurately represented by a two layer model or a three layer model. When a survey is conducted using an EM-38, the depth of penetration of the instrument is fairly shallow. In such a case a two layer model may often represent the soil situation in a field being surveyed, where the topsoil overlays the root zone. In such cases it is useful to determine the conductivity in the second layer. This conductivity may be correlated with other soil properties that are important for agricultural applications.

If a two-layer soil exists there are three unknown values:

σ_1	the conductivity of the first (top) layer
σ_2	the conductivity of the second layer
T	the thickness of the first layer

If we know these three values we can predict the apparent conductivity reading at any instrument height from the following formulas:

$$\sigma_{v,2} := \frac{\sigma 1\left(-\sqrt{4H^2+1}+\sqrt{4H^2+8HT+4T^2+1}\right)}{\sqrt{4H^2+8HT+4T^2+1}\cdot\sqrt{4H^2+1}} + \frac{\sigma 2}{\sqrt{4H^2+8HT+4T^2+1}} \quad (17)$$

$$\begin{aligned}\sigma_{h,2} := {} & \sigma 1\left(2T-\sqrt{4H^2+8HT+4T^2+1}+\sqrt{4H^2+1}\right) \\ & + \sigma 2\left(-2H-2T+\sqrt{4H^2+8HT+4T^2+1}\right)\end{aligned} \quad (18)$$

H is the height in normalized units. H=h/s, where h is in meters and s is the intercoil distance of the ground conductivity meter. For the EM-31 s=3.66 meters and for the EM-38 s=1 meter.

The thickness, T, is also in normalized units, where T = t/s, and t is the thickness in meters.

$\sigma_{v,2}$ is the apparent conductivity for a 2-layer model that would be measured by a ground conductivity meter in the vertical dipole position. $\sigma_{h,2}$ is the apparent conductivity that would be measured in the horizontal dipole position.

We want to calculate the values of σ_1, σ_2 and T from conductivity measurements taken at or near the surface. We can solve the above equations for these three unknown values if we have three separate conductivity measurements at any one location. There are several ways of acquiring three measurements at one location using a single instrument, including:

- Three vertical dipole readings taken with an EM-38 at different heights.
- Three horizontal dipole readings taken with an EM-38 at different heights.
- Three vertical dipole readings taken with an EM-31 at different heights.
- Three horizontal dipole readings taken with an EM-31 at different heights.

If we have more than one instrument available, there are several other ways we can acquire three measurements at one point. One possible combination is to include an EM-31 with an EM-38DD. In this case three readings are taken at each point: The EM-31 vertical reading, and the EM-38DD vertical and horizontal readings. If the two instruments are towed by a vehicle, we will have three apparent conductivity readings for each survey point. This is a major advantage over a single instrument situation where we would need to stop and take several measurements at one point in order to estimate conductivity values. With the three instrument method we can estimate conductivities with depth and topsoil thickness at every survey point.

There are several ways of setting up a three instrument survey using an EM-31 and EM-38DD:

- Both the EM-31 and EM-38DD are at the same height. The EM-31 is in the vertical dipole position.
- Both the EM-31 and EM-38DD are at the same height. The EM-31 is in the horizontal dipole position.
- Both the EM-31 and EM-38DD are at different heights. The EM-31 is in the vertical dipole position.
- Both the EM-31 and EM-38DD are at different heights. The EM-31 is in the horizontal dipole position.

Other types of instrument configurations are also possible. For example three EM-38s could be placed at different heights. The instruments could all be in the horizontal position, vertical position, or they could be a combination of horizontal and vertical positions.

We attempted to find analytic solutions for a variety of instrument combinations and instrument heights. All of the instrument configurations mentioned above have different values of instrument orientation, instrument height and intercoil spacing. As a result we expected that each unique combination of instruments and heights would result in a different set of analytic formulas for σ_1, σ_2 and T.

Methods

For each method we take three apparent conductivity readings at a location. We know the orientation, height and intercoil spacing of each instrument as well as three apparent conductivity readings. With these values we set up three simultaneous equations using the formulas we derived for the two layer case. We then solved these three equations

simultaneously and tried to find an analytic solution. If an analytic solution was not found, a numerical solution was attempted. In all cases the solutions were compared with the conductivity and thickness values entered into the two-layer earth model.

We also tried to solve the equations where one of the three model values was known. In many situations the depth to a second layer may be known from previous drilling, roadcuts or other features that provide a soil cross section where layers may be observed. In this case we solved two simultaneous equations for the unknown conductivities. We used various combinations of instruments and heights.

Results

We were unable to derive analytic solutions for the case where a single instrument is used at three different heights, or where multiple instruments (combinations of EM-31, EM-38, vertical and horizontal orientations) were used. This situation is common in electromagnetic inversion studies, and our results indicate that ground conductivity instruments suffer from the same problem as other electromagnetic methods.

We were able to use numerical methods to solve several of the three instrument configurations. We still need to evaluate how stable these methods are to errors in the data and in the starting values used to make the calculations

We were successful in calculating analytic solutions when the thickness (T) of the first layer is known. We found analytic solutions for the following instrument combinations (solutions are available in Appendix F):

- Two readings at different heights taken by the EM-38 in the vertical dipole position.
- Two readings at different heights taken by the EM-38 in the horizontal dipole position.
- Two readings at different heights taken by the EM-31 in the vertical dipole position.
- Two readings at different heights taken by the EM-31 in the horizontal dipole position.
- A vertical dipole and a horizontal dipole reading taken by an EM-38DD at ground level.
- A vertical dipole and a horizontal dipole reading taken by an EM-31 at ground level.
- A vertical dipole EM-31 and a vertical dipole EM-38 reading taken at different heights.
- A vertical dipole EM-31 and a horizontal dipole EM-38 reading taken at different heights.

Conclusions

If a two layer conductivity model is an accurate portrayal of subsurface conditions then it is possible to calculate the conductivity levels and the thickness of the top layer. In order to calculate these values three instruments must take readings at each point.

If only two instruments are available, such as two EM-38s, then analytic solutions can be found if the thickness of the first layer is known. In some cases this thickness is known from previous water wells that have been dug or from core samples.

Future Work

Numerical methods can be devised for additional combinations of three instruments. All of these numerical methods should be tested for computational stability given the inaccuracies in the initial guesses used to solve the numerical equations. In addition a sensitivity analysis should be carried out to determine which methods are most robust to data measurement errors.

All solutions, both analytic and numeric, should be field tested against core samples taken at the test sites.

Statistical method of estimating the conductivity of the top soil layer

Ground conductivity meters such as the Geonics EM-38 are used to measure apparent soil conductivity. Agriculturally useful soil properties can often be estimated from these conductivity readings. Many ground conductivity surveys use a dual dipole instrument, where there are two pairs of transmitting and receiving coils in different orientations. In such cases two estimates of conductivity are made. Each estimate is based on a different weighted average over depth.

If the soil consists of two layers with different conductivity, it is not possible to analytically calculate the soil conductivity values in each layer. There are three unknown variables: the conductivity values of each layer and the thickness of the top layer. There are only two known values: the readings of the two conductivity meters. To get around this problem we have devised a statistical method of predicting the conductivity values of each layer from the two instrument readings.

Methods

We constructed a conductivity model for a two-layered earth based on the analysis by McNeill (1980). We then created 20,000 random observations. For each observation the conductivities of the first and second layers were randomly assigned to the range of 10 to 500 mS/m. A maximum value of 500 mS/m was chosen since the Geonics EM-38 instrument underestimates the apparent conductivity when the apparent ground conductivity is above this value (McNeill 1980, Hendrickx et al. 2002). The thickness of the top layer was randomly assigned to the range of 0.1 to 5.0 meters. At a depth of 5 meters the relative response curve is measuring 90% of the conductivity below the surface as measured by the vertical dipole and 95% as measured by the horizontal dipole.

The apparent conductivity measured by the EM-38DD sensors in the both the vertical and horizontal dipole orientations were then calculated for each of the randomly created observations. This data set was then used to derive regression equations to estimate the values of σ_1 and σ_2, and T. σ_1 is the unknown conductivity of the top or first layer; σ_2 is the unknown conductivity of the second layer; and T is the thickness of the top layer.

We created several regression models with $\sigma_{app,V}$ and $\sigma_{app,H}$ as independent variables (the known values measured by the EM-38) and σ_1, σ_2 and T as dependent variables. The method of least squares was used to estimate the regression coefficients. We created various models with polynomial terms, other functions (e.g. powers and logarithms), as well as interaction terms.

Results

The regression equations provided good estimates of σ_1, and poor estimates of σ_2 and T. The following discussion deals solely with predictions of σ_1. A linear model (model 1) worked best. A preliminary analysis of our regression model indicated that the least squares estimates of σ_1 may have been affected by outliers. After deleting a number of

outliers the regression model was refit (model 2). After using both models to predict all original data points model 2 proved to be more accurate. For model 2, only 6.7% of the residuals were greater than 10 mS/m, as compared to 19.5% for model 1.

Model 2

The prediction equation for σ_l using model 2 is:

$$\sigma_l' = 3.943 - 1.237*\sigma_{app,V} + 2.222*\sigma_{app,H} \qquad (19)$$

The standard errors for each of the coefficients are:

Intercept	0.0767
$\sigma_{app,V}$	0.0016
$\sigma_{app,H}$	0.0015

$R^2 = 0.985$. This value was calculated using all data, including the outliers that had been deleted when creating model 2.

The residuals were not normally distributed. They are of an unknown distribution, meaning we cannot calculate confidence intervals or prediction intervals. However we can estimate how accurate a prediction is likely to be based on residuals for the entire set of 20,000 observations (Figure 36). 93% of the prediction errors are less than 10 mS/m. 4% of our predictions that have errors larger than 25 mS/m, and 2.18% that have errors larger than 50 mS/m. In rare cases (0.81%) the predictions will have errors larger than 100 mS/m. However, these errors only occur in cases where the thickness of the top soil layer is less than 0.5 m. For example, all errors larger than 25 mS/m occurred when T was smaller than 0.5 m. The histogram in Figure 37 shows the distribution of depth when the errors were larger than 25 mSm^{-1}.

Figure 36: Sigma1 Prediction Errors versus the predicted value

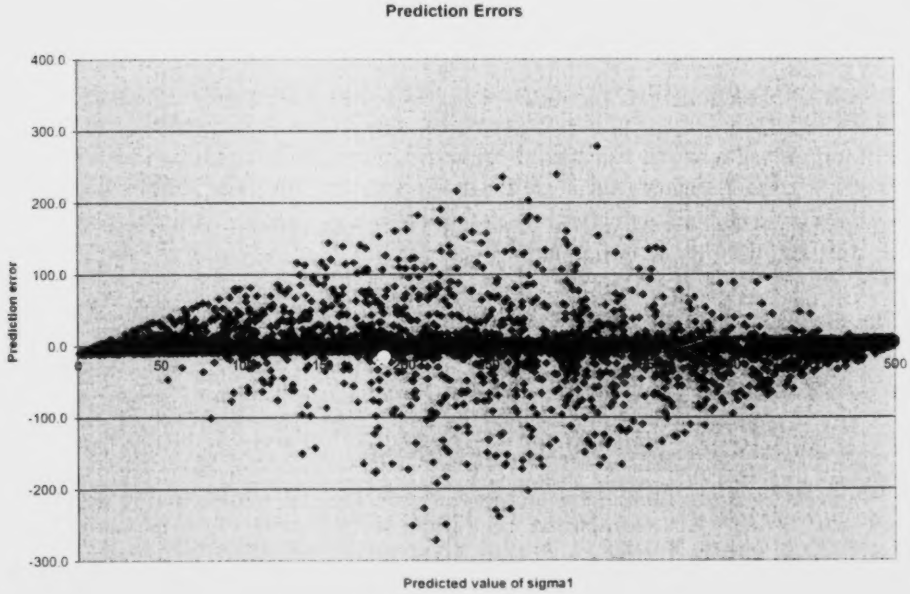
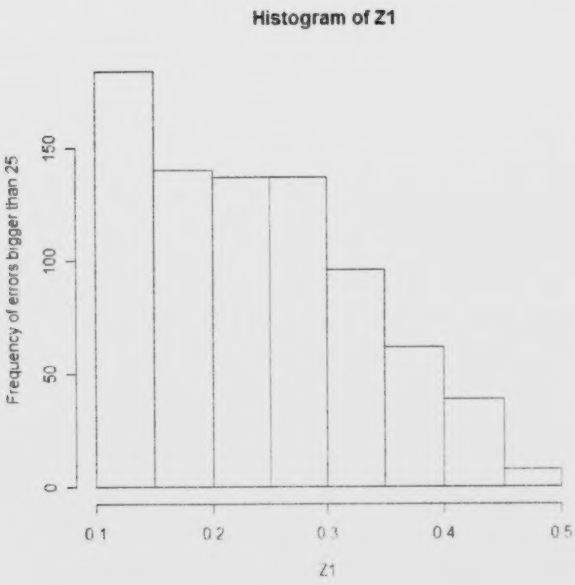


Figure 37: Frequency of prediction errors greater than $25mSm^{-1}$ as a function of the thickness of the first layer



Model 3

If we know that the topsoil thickness is large, we can construct a more accurate model to predict the values of $\sigma_{app,V}$. Given the smaller proportion of outliers with increasing thickness, our prediction equation should be more accurate if we derive it only from data based on observations where the thickness is large. In order to test this, we developed a new model (model 3) to use in the case where T is known to be greater than 0.5 meters. We fit a regression model using all the randomly created data points where T was between 0.5 and 5 meters. We used the same method to fit the regression model as we had earlier. We deleted a number of residuals that were affecting the least squares estimates, and then we refit the model.

For model 3 the prediction equation for σ_1 is:

$$\sigma_1' = 1.010 - 1.090*\sigma_{app,V} + 2.086*\sigma_{app,H} \qquad (20)$$

The standard errors for each of the coefficients are:

Intercept	0.0213
$\sigma_{app,V}$	0.0005
$\sigma_{app,H}$	0.0005

$R^2 = 0.999$

Prediction errors for this regression equation were small. The largest residual (prediction error) was 41 mS/m, and only 1% of the residuals were greater than 15 mS/m. Ninety-four percent of the residual errors were less than 3 mS/m.

Model 4

The magnitude of the prediction errors decreases with as the thickness of the top layer increases. If the top layer is known to have a thickness of 1 meter or more, the regression equation becomes:

$$\sigma_1' = 0.282 - 1.037*\sigma_{app,V} + 2.036*\sigma_{app,H} \qquad (21)$$

The standard errors for each of the coefficients are:

Intercept	0.0077
$\sigma_{app,V}$	0.0002
$\sigma_{app,H}$	0.0002

$R^2 = 0.999$

Many ground conductivity surveys are carried out with an EM-38DD conductivity meter. Such surveys provide two estimates of the apparent ground conductivity. Contour maps based on these apparent conductivity readings can highlight geographic changes in apparent conductivity, but do not provide much information about conductivity changes with depth. If the soil has uniform conductivity to the effective penetration depth of the conductivity meter, then the apparent conductivity measured by the vertical and

horizontal dipole instruments will be identical, and will be a measure of the apparent conductivity of the soil.

Since the apparent conductivity readings of the EM-38DD are a weighted average over depth, they do not represent the actual conductivity values of any soil layer. Knowledge of actual, rather than apparent conductivity values would allow us to make more accurate predictions of other soil properties.

By using a statistical method we can now make good predictions of the conductivity of the top layer of a two-layered soil profile. In many situations this two-layer model is a good approximation to the actual soil structure. While we cannot make accurate estimates of the thickness of the top layer or the conductivity of the second layer, we can determine from the $\sigma_{app.V} / \sigma_{app.H}$ ratio whether the second layer has a higher or lower conductivity than the top layer.

In some cases we may have additional information about soil layers and thickness from water wells, roadcuts and dugouts. In such cases we may be able to determine if a one or two layer model is appropriate. If two soil layers are present in the top 5 meters, we can also determine if the first layer has a thickness that is either greater than 0.5 meters or greater than 1 meter. In either case the appropriate prediction model can be used to estimate the conductivity in the top layer.

Conclusions

If soil conductivity is uniform down to the penetration depth of a ground conductivity meter, then a single instrument will provide an accurate reading of the soil conductivity. A quick check for uniform conductivity with depth can be made by taking two readings at the surface. One reading is taken with the instrument in the vertical dipole orientation and the other is taken with the instrument in the horizontal dipole orientation. If the ratio of the vertical to horizontal readings is between 0.95 and 1.05, then a uniform conductivity model will likely be a good fit to the data.

If the ratio is outside this range, then the conductivity can be modeled as a two or more layered earth. In the case of a two layered earth there are three unknown values: the conductivity of the two layers and the thickness of the top layer. If a dual conductivity meter such as the Geonics EM38DD is used to conduct a soil survey, then there are only two readings made at each location. There is insufficient information to provide an exact solution for the three unknowns. It is possible to estimate the conductivity using a statistical approach. Good estimates of the conductivity of the first layer can be made using a linear regression model based the values of the two Geonics conductivity meters. The accuracy of the predictions increases as the thickness of the first layer increases. It is not possible to accurately predict the thickness of the top layer or the conductivity of the second layer using a statistical approach. Other analytic methods are needed in order to do this.

Topographic Attributes

The Topographic Attributes (TA) program calculates several soil attributes from the elevation of a given area. In particular this program calculates several surface variables that affect hydrological parameters. The program reads the grid file of elevation readings created by the ETREMS program during a soil survey.

The computer program makes calculations for each point in the elevation grid by analyzing the 3 X 3 set of points surrounding and including the grid point. From this set of 9 points the program calculates various topographic parameters, such as the slope and curvature at that point. From this information the program can determine the direction of surface water flow through the area consisting of the 3 X 3 set of points. This process is carried out for every point in the grid representing the surveyed field.

Once these calculations have been carried out for the entire grid, the program calculates the proportion of water that flows from each grid square into the neighbouring grid squares. The program analyzes data from the grid points at the lowest elevation and moves upslope, calculating the contribution of each higher grid point to neighbouring lower-elevation grid points.

Once these calculations have been carried out for each grid point, the topographic wetness index is calculated for the grid cells. This index can be used to predict other soil attributes, such as topsoil depth.

Future Work

The various soil properties measured by the program, including the wetness index, should be correlated with production yields for various crops in order to determine how topography affects yield. The wetness index should also be tested as a useful guide to fertilizer application in fields that are not flat.

Recommendations

The addition of an EM-31 conductivity instrument can increase the amount of soil conductivity information with depth. The addition cost of surveying a field with a third instrument is only slightly higher than the cost of surveying without this instrument.

Conductivity surveys can be made easier by attaching the instruments to an ATV rather than towing them. Correction factors can be calculated for the effect of the ATV on conductivity readings.

EM-38 and EM-31 instruments can be oriented in several ways in order to reduce the amount of interference they create with each other.

The EM-38 and EM-31 instruments can be mounted above the surface of the earth and useful subsurface conductivity estimates can be made from these readings.

Increasing the instrument height will increase the depth penetration of the instruments. The instrument height can be set to meet the depth penetration requirements of a particular survey.

Interpretation techniques can be used to estimate soil conductivity changes with depth.

Future work

Specific recommendations for further research have been made in the various chapters dealing with each research problem. The recommendations presented in this section deal with more general problems that need to be addressed.

Determining the conductivity of soil with depth is not an easy problem to solve. We need improved interpretation techniques to determine how many soil layers there are in the top five meters, how thick each layer is, and how conductive each layer is. If we have accurate conductivity measurements with depth we will be able to make better estimates of soil moisture, salinity, and nutrient availability in the root zone.

Calculating soil conductivity in each layer is relatively easy if we know how many layers there are and how thick each layer is. We may be able to determine this information through the use of penetrometers. Penetrometers are simple instruments that can be pushed into the soil. Some penetrometers allow the surveyor to obtain a core sample without the need for drilling. The surveyor can examine this core and determine how thick each layer is. If these layers correspond to conductivity differences, then accurate conductivity values can be calculated for each layer.

Another type of penetrometer measures soil conductivity directly. Conductivity is measured as the instrument is pushed deeper into the soil. This type of instrument allows us to determine the thickness of each conductive layer, and provides us with soil conductivity values that can be correlated with measurements taken at the surface. Conductivity measurements made in this manner are much cheaper drilling soil cores and then shipping the cores to a laboratory where the conductivity is measured for different soil depths.

We would like to determine if we can reduce soil sampling costs by using penetrometers instead of or as a complementary method to traditional soil coring and lab analysis techniques.

If we do not know the thicknesses of the various layers then numerical techniques must be used to estimate conductivity changes with depth. We need to develop more accurate numerical methods.

Another promising area for research is to compare crop yields with conductivity levels at the field level. Various soil properties affect yield, such as soil nutrients, soil moisture levels and salinity levels. Since conductivity is dependent on many of these factors, it may be a good estimator of crop yield within a field. We would like to determine how well conductivity estimates each of these soil properties in addition to the yield.

Changes from the original research plan

We were unable to test the accuracy of some of our interpretation methods to the extent we required. In particular we were unable to test how accurately we could map subsurface soil-moisture conditions and contaminant extent.

As part of our research project we developed a computer program that allows soil surveyors to measure conductivity from three instruments simultaneously. We completed this program at the end of August. We successfully tested the program using PFRA's equipment. However this completion time coincided with a busy period for PFRA. PFRA conducts a number of soil conductivity surveys during the short period after crops have been harvested and before the weather makes surveying impossible. While PFRA staff members were extremely helpful in making their equipment available to us, they had prior commitments during this period which limited the number of sites we could survey.

We were able to reduce the impact of this problem by obtaining conductivity survey results and core sample logs from the Saskatchewan Ministry of Agriculture's Irrigation Development Branch in Outlook. This allowed us to test our interpretation methods for conductivity surveys made with two instruments.

In hindsight this problem could have been overcome if we had carried the research out over a two year rather than a one year period.

As the research progressed we realized that more soil conductivity information could be obtained by re-interpreting the data from existing two-instrument surveys. While this was not one of our original research objectives, we developed and successfully tested new interpretation techniques for such surveys.

We also created a fast contouring program that can be used in the field. Originally we planned to create a computer program to capture data from three conductivity instruments. This conductivity data would be taken back to the lab where it would be analyzed. The data would be transferred to other computers running GIS software that could be used to create contour maps. As we developed this software, survey operators indicated to us that it would be useful if they could see the conductivity results before they left the survey site. In this way the operator can determine if there are areas in the field that need greater coverage, and can continue the survey from any location in the field. As a result we incorporated a fast contouring program into the data capture software.

Another deliverable we provided was a computer program to create topographic attribute maps from GPS altitude data. These maps may be useful in estimating various hydrological parameters such as soil moisture and water flow direction. The development of the program was not included in our original objectives.

Appendix A
Interference between the EM-31 and EM-38DD instruments

Parallel instruments

Table 1

Orientation 1 – Transmitter-receiver reversed

EM 31 readings - 31 stationary; 38 moves

Separation (m)	Conductivity (mS/m)
0.2	127
0.4	126.5
0.6	126.5
0.8	126
1.0	126.5
1.2	126.5
1.4	126
1.8	126
2.0	126
2.5	126
3.0	126
3.8	126

Table 2

Orientation 2 – Transmitter-receiver parallel

EM 31 readings - 31 stationary; 38 moves

Separation (m)	Conductivity (mS/m)
0.2	126
0.6	126.5
0.8	126
1.0	126

Table 3

Orientation 1 – Transmitter-receiver reversed

EM 38 readings - 38 stationary; 31 moves

Separation (m)	Vertical	Horizontal
0.40	63	39
0.60	79	54
0.80	82	60
1.00	81	60
1.40	80	60
1.60	79	60.0
2.00	78	59.5
2.45	78	60.0
3.42	77	60.0

Perpendicular instruments

Effect of the EM-38DD on EM-31 readings

Table 4

Orientation 1 – T31-T38

EM 31 readings - 31 stationary; 38 moves

Separation (m)	Conductivity (mS/m)
0.30	123
0.49	123
0.75	123.5
1.00	123
1.15	123

Table 5

Orientation 1 – R31-R38

EM 31 readings - 31 stationary; 38 moves

Separation (m)	Conductivity (mS/m)
0.25	122
0.40	123
0.50	123.5
0.65	124
1.08	124

Table 6

Orientation 1 – T31-R38

EM 31 readings - 31 stationary; 38 moves

Separation (m)	Conductivity (mS/m)
0.30	123
0.52	123
0.76	123
1.08	123.5

Table 7

Orientation 1 – R31-T38

EM 31 readings - 31 stationary; 38 moves

Separation (m)	Conductivity (mS/m)
0.35	176
0.59	133
0.74	121
0.90	123
1.15	124

Effect of the EM-31 on EM-38DD readings

Table 8

Orientation 1 – T31-T38

EM 38 readings - 38 stationary; 31 moves

Separation (m)	Vertical	Horizontal
0.22	49	40
0.50	53	43.5
0.72	55.5	45.5
1.04	60	42
1.31	62	40.5

Table 9

Orientation 2 – R31-T38

EM 38 readings - 38 stationary; 31 moves

Separation (m)	Vertical	Horizontal
0.37	65	41
0.54	65	41
0.78	65	40.5
0.91	65	41
1.28	65	40.5

Appendix B
Attaching the EM-38DD to an ATV

Site 1

Height	V-manual	H - manual	V-boom	H-boom
0	86	56		
9			85.5	65
15			82	62
16.73	76	46		
21			78	58.5
27			74	55.5
31.65	67	39		
34			71	53.5
42			66	50
46.53	59	34		
50			63	48
56			60.5	46.5
61.75	54	30		

Site 2

Height	V-manual	H - manual	V-boom	H-boom
0	74	46		
9			75.5	56.5
16.73	64	38		
18.4			70	52
24.8			67	50
30			63	47
31.65	57	32		
36			61	45.5
46.53	50	28		
47			57	43
61.75	43	25		

Site 3

Height	V-manual	H - manual	V-boom	H-boom
0	47	29		
9			52	41.5
16.4			49.5	40.5
16.73	42	24		
26			47	38.5
31.6			46	37
31.65	37	21		
38			44	36.5
46.53	34	19		
47			42	34.5
61.75	29	17		

Site 4

Height	V-manual	H - manual	V-boom	H-boom
0	86	54		
9			87	65
16			83	61.5
16.73	74	44		
21			79	58.5
28.1			75	56
31.65	65	38		
33			73	54
41			68	51
46.53	58	33		
49			65	49
61.75	52	29		

Site 5

Height	V-manual	H - manual	V-boom	H-boom
0	38	24		
9			43	37.5
11			42	36.5
16			41.5	35.5
16.73	35	20		
19			40.5	35
25			39	34.5
31			38	33.5
31.65	32	18		
37			37	32.5
43			36	32
46.53	29	16		
50			35	31.5
56.5			35	31.5
61.75	27	14		

Appendix C
Attaching the EM-31 to an ATV

EM31 is towed behind the ATV

Vertical conductivity measurements when the EM-31 is towed behind the ATV.
Distance is between the centre of the ATV and the centre of EM-31 (receiver forward)
Measured conductivity when no ATV is present is 126 mS/m

Distance (m)	Conductivity (mS/m)
1.47	194
2.28	173
3.28	136
4.00	130
4.93	128
6.40	127

Case 1: the centre of the ATV is aligned with the centre of the EM-31

		Conductivity measurement (mS/m)			Percent error		
Height	No ATV	Tf 0.34 m	Tf 0.78 m	Rf 0.78 m	Tf 0.34 m	Tf 0.78 m	Rf 0.78 m
0.62	107	150	134	135	40%	18%	21%
0.76	101	139	125	127	38%	17%	21%
0.92	95	127	115	119	34%	16%	21%
1.07	90	113	105	108	26%	13%	17%

Case 2: the centre of the EM-31 is displaced from the centre of the ATV; ΔX = 0.34m

Height	No ATV	Tf Y=0.0m	Tf Y=0.40m	Tf Y=0.70m	Rf Y=0.18m	Rf Y=0.34m	Rf Y=0.48m	Rf Y=0.67m
0.62	107	150	151	154	151	151	154	154
0.76	101	139	143	146	143	142	145	149
0.92	95	127	125	125	130	133	132	136
1.07	90	113	112	112	114	118	115	118

Case 3: the centre of the EM-31 is displaced from the centre of the ATV; ΔX = 0.78m

Height	No ATV	Tf Y=0.0m	Tf Y=0.33m	Tf Y=0.72m	Tf Y=1.08m	Rf Y=0.0m	Rf Y=0.33m	Rf Y=0.72m	Rf Y=1.08m
0.62	107	134	134	135	131	135	136	135	136
0.76	101	125	125	127	123	127	127	128	130
0.92	95	115	114	112	111	119	118	119	117
1.07	90	105	105	102	100	108	109	110	109

Appendix D
Field tests of EM-38 instrument height correction formula

Site 1

Height	V	H	Vertical Corrected	Vertical % Error	Horizontal Corrected	Horizontal % Error
0	86	56	86	0%	56	0%
0.165	76	46	80	-7%	64	14%
0.315	67	39	79	-8%	71	26%
0.464	59	34	80	-6%	78	39%
0.616	54	30	86	0%	85	51%
0.763	49	27	89	4%	90	62%
0.916	45	24	94	9%	94	68%
1.065	43	22	101	18%	99	76%
1.212	39	21	102	19%	106	89%
1.364	37	18	108	25%	101	81%
1.513	35	16	112	30%	99	78%

Site 2

Height	V	H	Vertical Corrected	Vertical % Error	Horizontal Corrected	Horizontal % Error
0	74	46	74	0%	46	0%
0.165	64	38	67	-9%	53	14%
0.315	57	32	67	-9%	58	26%
0.464	50	28	68	-8%	64	40%
0.616	43	25	68	-8%	70	53%
0.763	40	22	73	-1%	74	60%
0.916	35	20	73	-1%	78	70%
1.065	32	18	75	2%	81	75%
1.212	31	17	81	10%	86	86%
1.364	28	15	81	10%	85	84%
1.513	25	14	80	8%	87	89%

Site 3

Height	V	H	Vertical Corrected	Vertical % Error	Horizontal Corrected	Horizontal % Error
0	47	29	47	0%	29	0%
0.165	42	24	44	-6%	33	14%
0.315	37	21	44	-7%	38	31%
0.464	34	19	46	-1%	44	50%
0.616	29	17	46	-2%	48	65%
0.763	27	15	49	5%	50	73%
0.916	26	13	54	15%	51	76%
1.065	21	12	49	5%	54	86%
1.212	21	11	55	17%	56	91%
1.364	19	10	55	17%	56	94%
1.513	18	10	57	22%	62	114%

Site 4

Height	V	H	Vertical Corrected	Vertical % Error	Horizontal Corrected	Horizontal % Error
0	86	54	86	0%	54	0%
0.165	74	44	78	-9%	61	13%
0.315	65	38	77	-11%	69	28%
0.464	58	33	79	-8%	76	40%
0.616	52	29	83	-4%	82	51%
0.763	47	26	86	0%	87	61%
0.916	43	23	90	4%	90	67%
1.065	40	21	94	9%	94	74%
1.212	36	19	94	10%	96	78%
1.364	33	17	96	11%	96	77%
1.513	31	16	99	15%	99	84%

Site 5

Height	V	H	Vertical Corrected	Vertical % Error	Horizontal Corrected	Horizontal % Error
0	38	24	38	0%	24	0%
0.165	35	20	37	-3%	28	15%
0.315	32	18	38	0%	33	36%
0.464	29	16	40	4%	37	53%
0.616	27	14	43	13%	39	64%
0.763	25	15	46	20%	50	109%
0.916	24	13	50	32%	51	112%
1.065	22	12.5	52	36%	56	133%
1.212	21	11	55	45%	56	131%
1.364	20	10	58	53%	56	135%
1.513	19	9	61	59%	56	133%

Appendix E
Field tests of EM-31 instrument height correction formula

Site 1

Height	V	H	Vertical Corrected	Vertical % Error	Horizontal Corrected	Horizontal % Error
0	125	121	125	0%	121	0%
0.165	123	113	124	-1%	124	2%
0.315	120	104	122	-3%	123	2%
0.464	116	95	120	-4%	122	1%
0.616	112	86.5	118	-5%	120	-1%
0.763	108	79	117	-6%	118	-2%
0.916	102	73	114	-9%	118	-2%
1.065	98	67	113	-9%	116	-4%
1.212	93	62	112	-11%	115	-5%
1.364	88	56	110	-12%	111	-8%
1.513	84	53	109	-13%	112	-7%

Site 2

Height	V	H	Vertical Corrected	Vertical % Error	Horizontal Corrected	Horizontal % Error
0	75	64.5	75	0%	65	0%
0.165	74	60	74	-1%	66	2%
0.315	72	56	73	-3%	66	3%
0.464	70	52	72	-4%	67	4%
0.616	67	48	71	-6%	67	3%
0.763	64	44	69	-8%	66	2%
0.916	61	40	68	-9%	65	0%
1.065	59	38	68	-9%	66	2%
1.212	56	35	67	-10%	65	1%
1.364	54	32	67	-10%	64	-1%
1.513	51	31	66	-12%	66	2%

Appendix F
Solutions to equations when the depth to the first layer is known

NOTATION

Known values at any point:

H1: Height of first instrument in units of h1/s, where h1 is the height in meters and s is the intercoil spacing of the instrument.

H2: Height of second instrument in units of h2/s, where h is the height in meters and s is the intercoil spacing of the instrument.

T: The thickness of the first layer in units of t/s, where t is the thickness in meters and s is the intercoil spacing of the instrument.

$\sigma_{V,H1}$ The measurement taken by the vertical dipole instrument at height 1.

$\sigma_{V,H2}$ The measurement taken by the vertical dipole instrument at height 2.

$\sigma_{h,H1}$ The measurement taken by the horizontal dipole instrument at height 1.

$\sigma_{h,H2}$ The measurement taken by the horizontal dipole instrument at height 1.

Unknown values that are solved for:

σ1 The conductivity of the first (top) layer.

σ2 The conductivity of the second layer.

Two vertical dipole readings are made at different heights with either the EM-38 or the EM-31

$$
\begin{aligned}
\sigma 1 = &-\sqrt{4\,H_1^{\,2}+1}\,\Big(-4\,\sigma_{V,H2}\sqrt{4\,H_2^{\,2}+8\,H_2\,T+4\,T^2+1}\,H_2^{\,2} \\
&-\sigma_{V,H2}\sqrt{4\,H_2^{\,2}+8\,H_2\,T+4\,T^2+1}+4\,\sigma_{V,H1}\sqrt{4\,H_1^{\,2}+8\,H_1\,T+4\,T^2+1}\,H_2^{\,2} \\
&+\sigma_{V,H1}\sqrt{4\,H_1^{\,2}+8\,H_1\,T+4\,T^2+1}\Big)\Big/\Big(-4\sqrt{4\,H_1^{\,2}+8\,H_1\,T+4\,T^2+1}\,H_2^{\,2} \\
&-\sqrt{4\,H_1^{\,2}+8\,H_1\,T+4\,T^2+1} \\
&+\sqrt{4\,H_2^{\,2}+1}\sqrt{4\,H_1^{\,2}+1}\sqrt{4\,H_2^{\,2}+8\,H_2\,T+4\,T^2+1}\Big)
\end{aligned}
$$

$$
\begin{aligned}
\sigma 2 = &-\sqrt{4\,H_2^{\,2}+1}\,\Big(\\
&\sigma_{V,H2}\sqrt{4\,H_2^{\,2}+8\,H_2\,T+4\,T^2+1}\sqrt{4\,H_2^{\,2}+1}\sqrt{4\,H_1^{\,2}+8\,H_1\,T+4\,T^2+1} \\
&-\sqrt{4\,H_1^{\,2}+1}\,\sigma_{V,H2}\sqrt{4\,H_2^{\,2}+8\,H_2\,T+4\,T^2+1}\sqrt{4\,H_2^{\,2}+1} \\
&+\sigma_{V,H1}\sqrt{4\,H_1^{\,2}+8\,H_1\,T+4\,T^2+1}\sqrt{4\,H_1^{\,2}+1}\sqrt{4\,H_2^{\,2}+1} \\
&-\sqrt{4\,H_1^{\,2}+1}\sqrt{4\,H_2^{\,2}+8\,H_2\,T+4\,T^2+1}\,\sigma_{V,H1}\sqrt{4\,H_1^{\,2}+8\,H_1\,T+4\,T^2+1}\Big)\Big/\Big(\\
&-4\sqrt{4\,H_1^{\,2}+8\,H_1\,T+4\,T^2+1}\,H_2^{\,2}-\sqrt{4\,H_1^{\,2}+8\,H_1\,T+4\,T^2+1} \\
&+\sqrt{4\,H_2^{\,2}+1}\sqrt{4\,H_1^{\,2}+1}\sqrt{4\,H_2^{\,2}+8\,H_2\,T+4\,T^2+1}\Big)
\end{aligned}
$$

Two horizontal dipole readings are made at different heights with either the EM-38 or the EM-31

$$\sigma 1 = (2 H_1 \sigma_{h,H2} + 2 T \sigma_{h,H2} - \sqrt{4 H_1^2 + 8 H_1 T + 4 T^2 + 1}\, \sigma_{h,H2} - 2 \sigma_{h,H1} H_2 - 2 \sigma_{h,H1} T$$
$$+ \sigma_{h,H1} \sqrt{4 H_2^2 + 8 H_2 T + 4 T^2 + 1}\,) \;/\; (4 H_1 T - 2 H_1 \sqrt{4 H_2^2 + 8 H_2 T + 4 T^2 + 1}$$
$$+ 2 H_1 \sqrt{4 H_2^2 + 1} + 2 T \sqrt{4 H_2^2 + 1} - \sqrt{4 H_1^2 + 8 H_1 T + 4 T^2 + 1}\, \sqrt{4 H_2^2 + 1}$$
$$- 4 H_2 T + 2 \sqrt{4 H_1^2 + 8 H_1 T + 4 T^2 + 1}\, H_2 - 2 \sqrt{4 H_1^2 + 1}\, H_2 - 2 \sqrt{4 H_1^2 + 1}\, T$$
$$+ \sqrt{4 H_1^2 + 1}\, \sqrt{4 H_2^2 + 8 H_2 T + 4 T^2 + 1}\,)$$

$$\sigma 2 = -(2 \sigma_{h,H1} T - \sigma_{h,H2} \sqrt{4 H_1^2 + 1} + \sqrt{4 H_1^2 + 8 H_1 T + 4 T^2 + 1}\, \sigma_{h,H2} - 2 T \sigma_{h,H2}$$
$$+ \sqrt{4 H_2^2 + 1}\, \sigma_{h,H1} - \sigma_{h,H1} \sqrt{4 H_2^2 + 8 H_2 T + 4 T^2 + 1}\,) \;/\; (4 H_1 T$$
$$- 2 H_1 \sqrt{4 H_2^2 + 8 H_2 T + 4 T^2 + 1} + 2 H_1 \sqrt{4 H_2^2 + 1} + 2 T \sqrt{4 H_2^2 + 1}$$
$$- \sqrt{4 H_1^2 + 8 H_1 T + 4 T^2 + 1}\, \sqrt{4 H_2^2 + 1} - 4 H_2 T$$
$$+ 2 \sqrt{4 H_1^2 + 8 H_1 T + 4 T^2 + 1}\, H_2 - 2 \sqrt{4 H_1^2 + 1}\, H_2 - 2 \sqrt{4 H_1^2 + 1}\, T$$
$$+ \sqrt{4 H_1^2 + 1}\, \sqrt{4 H_2^2 + 8 H_2 T + 4 T^2 + 1}\,)$$

A vertical dipole reading and a horizontal dipole reading are made at different heights. The instrument may be either the EM-38 or the EM-31.

$$
\begin{aligned}
\sigma 1 = (& 8\,T\sigma_{v,H1}\sqrt{4\,H_1^2+8\,H_1\,T+4\,T^2+1}\;H_1^2+2\,T\sigma_{v,H1}\sqrt{4\,H_1^2+8\,H_1\,T+4\,T^2+1} \\
& +8\,H_2\,\sigma_{v,H1}\sqrt{4\,H_1^2+8\,H_1\,T+4\,T^2+1}\;H_1^2 \\
& +2\,H_2\,\sigma_{v,H1}\sqrt{4\,H_1^2+8\,H_1\,T+4\,T^2+1} \\
& -4\sqrt{4\,H_2^2+8\,H_2\,T+4\,T^2+1}\;\sigma_{v,H1}\sqrt{4\,H_1^2+8\,H_1\,T+4\,T^2+1}\;H_1^2 \\
& -\sqrt{4\,H_2^2+8\,H_2\,T+4\,T^2+1}\;\sigma_{v,H1}\sqrt{4\,H_1^2+8\,H_1\,T+4\,T^2+1}+4\,\sigma_{h,H2}\,H_1^2 \\
& +\sigma_{h,H2})\;/\;(-8\,H_2\,H_1^2-2\,H_2+4\sqrt{4\,H_2^2+1}\;H_1^2+\sqrt{4\,H_2^2+1} \\
& +2\sqrt{4\,H_1^2+8\,H_1\,T+4\,T^2+1}\sqrt{4\,H_1^2+1}\;T \\
& +2\sqrt{4\,H_1^2+8\,H_1\,T+4\,T^2+1}\sqrt{4\,H_1^2+1}\;H_2 \\
& -\sqrt{4\,H_1^2+8\,H_1\,T+4\,T^2+1}\sqrt{4\,H_1^2+1}\sqrt{4\,H_2^2+8\,H_2\,T+4\,T^2+1}\;)
\end{aligned}
$$

$$
\begin{aligned}
\sigma 2 = (& 8\,T\sigma_{v,H1}\sqrt{4\,H_1^2+8\,H_1\,T+4\,T^2+1}\;H_1^2+2\,T\sigma_{v,H1}\sqrt{4\,H_1^2+8\,H_1\,T+4\,T^2+1} \\
& -4\sqrt{4\,H_2^2+8\,H_2\,T+4\,T^2+1}\;\sigma_{v,H1}\sqrt{4\,H_1^2+8\,H_1\,T+4\,T^2+1}\;H_1^2 \\
& -\sqrt{4\,H_2^2+8\,H_2\,T+4\,T^2+1}\;\sigma_{v,H1}\sqrt{4\,H_1^2+8\,H_1\,T+4\,T^2+1}+4\,\sigma_{h,H2}\,H_1^2 \\
& +\sigma_{h,H2}-\sqrt{4\,H_1^2+1}\sqrt{4\,H_1^2+8\,H_1\,T+4\,T^2+1}\;\sigma_{h,H2} \\
& +4\,\sigma_{v,H1}\sqrt{4\,H_1^2+8\,H_1\,T+4\,T^2+1}\sqrt{4\,H_2^2+1}\;H_1^2 \\
& +\sigma_{v,H1}\sqrt{4\,H_1^2+8\,H_1\,T+4\,T^2+1}\sqrt{4\,H_2^2+1}\;)\;/\;(-8\,H_2\,H_1^2-2\,H_2 \\
& +4\sqrt{4\,H_2^2+1}\;H_1^2+\sqrt{4\,H_2^2+1}+2\sqrt{4\,H_1^2+8\,H_1\,T+4\,T^2+1}\sqrt{4\,H_1^2+1}\;T \\
& +2\sqrt{4\,H_1^2+8\,H_1\,T+4\,T^2+1}\sqrt{4\,H_1^2+1}\;H_2 \\
& -\sqrt{4\,H_1^2+8\,H_1\,T+4\,T^2+1}\sqrt{4\,H_1^2+1}\sqrt{4\,H_2^2+8\,H_2\,T+4\,T^2+1}\;)
\end{aligned}
$$

If both readings are made at the surface. the solutions become:

$$\sigma 1 = \frac{1}{2} \frac{\sigma_{h,0} + 2\,T\,\sigma_{v,0}\sqrt{4\,T^2+1} - 4\,\sigma_{v,0}\,T^2 - \sigma_{v,0}}{T\,(-2\,T + \sqrt{4\,T^2+1}\,)}$$

$$\sigma 2 = \frac{1}{2} \frac{\sigma_{h,0} + 2\,T\,\sigma_{v,0}\sqrt{4\,T^2+1} - 4\,\sigma_{v,0}\,T^2 - \sigma_{v,0} - \sqrt{4\,T^2+1}\,\sigma_{h,0} + \sigma_{v,0}\sqrt{4\,T^2+1}}{T\,(-2\,T + \sqrt{4\,T^2+1}\,)}$$

An EM-31 vertical dipole reading and an EM-38 vertical dipole reading made at different heights.

$$
\begin{aligned}
\sigma 1 = - & \sqrt{4\,h_{31}^{\;2} + 13.3956}\;\Big(4\,\sigma_{V,H1}\sqrt{4\,h_{38}^{\;2} + 8\,h_{38}\,t + 4\,t^2 + 1}\;h_{38}^{\;2} \\
& + \sigma_{V,H1}\sqrt{4\,h_{38}^{\;2} + 8\,h_{38}\,t + 4\,t^2 + 1} - 4\,\sigma_{V,H2}\sqrt{4\,h_{31}^{\;2} + 8\,h_{31}\,t + 4\,t^2 + 13.3956}\;h_{38}^{\;2} \\
& - \sigma_{V,H2}\sqrt{4\,h_{31}^{\;2} + 8\,h_{31}\,t + 4\,t^2 + 13.3956}\;\Big) \Big/ \Big(\\
& 4\sqrt{4\,h_{31}^{\;2} + 8\,h_{31}\,t + 4\,t^2 + 13.3956}\;h_{38}^{\;2} + \sqrt{4\,h_{31}^{\;2} + 8\,h_{31}\,t + 4\,t^2 + 13.3956} \\
& - \sqrt{4\,h_{38}^{\;2} + 1}\sqrt{4\,h_{31}^{\;2} + 13.3956}\sqrt{4\,h_{38}^{\;2} + 8\,h_{38}\,t + 4\,t^2 + 1}\;\Big)
\end{aligned}
$$

$$
\begin{aligned}
\sigma 2 = \Big(\Big(\sigma_{V,H1} & \sqrt{4\,h_{38}^{\;2} + 8\,h_{38}\,t + 4\,t^2 + 1}\sqrt{4\,h_{38}^{\;2} + 1}\sqrt{4\,h_{31}^{\;2} + 8\,h_{31}\,t + 4\,t^2 + 13.3956} \\
& - \sqrt{4\,h_{31}^{\;2} + 13.3956}\;\sigma_{V,H1}\sqrt{4\,h_{38}^{\;2} + 8\,h_{38}\,t + 4\,t^2 + 1}\sqrt{4\,h_{38}^{\;2} + 1} \\
& + \sigma_{V,H2}\sqrt{4\,h_{31}^{\;2} + 8\,h_{31}\,t + 4\,t^2 + 13.3956}\sqrt{4\,h_{31}^{\;2} + 13.3956}\sqrt{4\,h_{38}^{\;2} + 1} - \\
& \sqrt{4\,h_{31}^{\;2} + 13.3956}\sqrt{4\,h_{38}^{\;2} + 8\,h_{38}\,t + 4\,t^2 + 1}\;\sigma_{V,H2} \\
& \sqrt{4\,h_{31}^{\;2} + 8\,h_{31}\,t + 4\,t^2 + 13.3956}\;\Big)\sqrt{4\,h_{38}^{\;2} + 1}\;\Big) \Big/ \Big(\\
& 4\sqrt{4\,h_{31}^{\;2} + 8\,h_{31}\,t + 4\,t^2 + 13.3956}\;h_{38}^{\;2} + \sqrt{4\,h_{31}^{\;2} + 8\,h_{31}\,t + 4\,t^2 + 13.3956} \\
& - \sqrt{4\,h_{38}^{\;2} + 1}\sqrt{4\,h_{31}^{\;2} + 13.3956}\sqrt{4\,h_{38}^{\;2} + 8\,h_{38}\,t + 4\,t^2 + 1}\;\Big)
\end{aligned}
$$

An EM-31 vertical dipole reading and an EM-38 horizontal dipole reading made at different heights.

$$
\begin{aligned}
\sigma 1 = {} & \sqrt{4\,h_{31}^{\,2} + 13.3956}\,\Big(-2\,h_{38}\,\sigma_{V,\,H2}\,\sqrt{4\,h_{31}^{\,2} + 8\,h_{31}\,t + 4\,t^2 + 13.3956} \\
& - 2\,t\,\sigma_{V,\,H2}\,\sqrt{4\,h_{31}^{\,2} + 8\,h_{31}\,t + 4\,t^2 + 13.3956} \\
& + \sqrt{4\,h_{38}^{\,2} + 8\,h_{38}\,t + 4\,t^2 + 1}\;\sigma_{V,\,H2}\,\sqrt{4\,h_{31}^{\,2} + 8\,h_{31}\,t + 4\,t^2 + 13.3956} - \sigma_{H,\,H1}\Big) \Big/ \Big(\\
& 2\,h_{38}\,\sqrt{4\,h_{31}^{\,2} + 13.3956} - 2\,h_{38}\,\sqrt{4\,h_{31}^{\,2} + 8\,h_{31}\,t + 4\,t^2 + 13.3956} \\
& - 2\,t\,\sqrt{4\,h_{31}^{\,2} + 8\,h_{31}\,t + 4\,t^2 + 13.3956} \\
& + \sqrt{4\,h_{38}^{\,2} + 8\,h_{38}\,t + 4\,t^2 + 1}\,\sqrt{4\,h_{31}^{\,2} + 8\,h_{31}\,t + 4\,t^2 + 13.3956} \\
& - \sqrt{4\,h_{38}^{\,2} + 1}\,\sqrt{4\,h_{31}^{\,2} + 13.3956}\,\Big)
\end{aligned}
$$

$$
\begin{aligned}
\sigma 2 = {} & \Big(-\sigma_{V,\,H2}\,\sqrt{4\,h_{31}^{\,2} + 8\,h_{31}\,t + 4\,t^2 + 13.3956}\,\sqrt{4\,h_{31}^{\,2} + 13.3956}\,\sqrt{4\,h_{38}^{\,2} + 1} \\
& - 2\,\sqrt{4\,h_{31}^{\,2} + 13.3956}\;t\,\sigma_{V,\,H2}\,\sqrt{4\,h_{31}^{\,2} + 8\,h_{31}\,t + 4\,t^2 + 13.3956} \; + \\
& \sqrt{4\,h_{31}^{\,2} + 13.3956}\,\sqrt{4\,h_{38}^{\,2} + 8\,h_{38}\,t + 4\,t^2 + 1}\;\sigma_{V,\,H2} \\
& \sqrt{4\,h_{31}^{\,2} + 8\,h_{31}\,t + 4\,t^2 + 13.3956} - \sigma_{H,\,H1}\,\sqrt{4\,h_{31}^{\,2} + 13.3956} \\
& + \sqrt{4\,h_{31}^{\,2} + 8\,h_{31}\,t + 4\,t^2 + 13.3956}\;\sigma_{H,\,H1}\Big) \Big/ \Big(2\,h_{38}\,\sqrt{4\,h_{31}^{\,2} + 13.3956} \\
& - 2\,h_{38}\,\sqrt{4\,h_{31}^{\,2} + 8\,h_{31}\,t + 4\,t^2 + 13.3956} - 2\,t\,\sqrt{4\,h_{31}^{\,2} + 8\,h_{31}\,t + 4\,t^2 + 13.3956} \\
& + \sqrt{4\,h_{38}^{\,2} + 8\,h_{38}\,t + 4\,t^2 + 1}\,\sqrt{4\,h_{31}^{\,2} + 8\,h_{31}\,t + 4\,t^2 + 13.3956} \\
& - \sqrt{4\,h_{38}^{\,2} + 1}\,\sqrt{4\,h_{31}^{\,2} + 13.3956}\,\Big)
\end{aligned}
$$

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